Determining the Dark Matter Relic Density in the mSUGRA $\tilde{\tau}$-$\tilde{\chi}_1^0$ Co-Annhiliation Region at the LHC

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We examine the stau-neutralino co-annihilation mechanism of the early universe at the LHC. We use minimal supergravity (mSUGRA) model and show that one can predict the dark matter relic density with an uncertainty of 6% with 30 fb^{-1}, which is comparable to the direct measurement by WMAP. This is possible by introducing measurements involving the b-quark jets to determine the mSUGRA parameters A_0 and $\tan\beta$ without requiring direct measurements of the stop and sbottom masses. Our methods provide a precision mass measurements of the gauginos, squark, and lighter stau without the mSUGRA assumption. These techniques can also be used in other regions of SUGRA parameter space.

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One of the important aspects of supersymmetry (SUSY), particularly when it is combined with supergravity grand unification (SUGRA GUTS) [1, 2] is that it resolves a number of the problems inherent in the Standard Model (SM). Thus, aside from solving the gauge hierarchy problem and predicting grand unification at the GUT scale $M_{\rm G} \sim 10^{16}$ GeV, which was subsequently verified at LEP [3], SUGRA GUTS allows for the spontaneous breaking of SUGRA at the $M_{\rm G}$ scale in a hidden sector, leading to an array of soft breaking masses. The renormalization group equations (RGEs) then show that this breaking of SUGRA leads naturally to the breaking of SU(2)×U(1) of the SM at the electroweak scale. This gives rise to an array of SUSY particles accessible to the Large Hadron Collider (LHC) as well as the contribute to a variety of phenomena testable at the electroweak scale, *e.g.*, the anomalous muon magnetic moment $g-2$ (which currently shows a 3.3σ effect [4]).

An additional feature of SUSY is that models with R-parity invariance automatically give rise to a cold dark matter (CDM) candidate, which is generally the lightest neutralino ($\tilde{\chi}_1^0$), implying a close connection between particle physics and early universe cosmology. The LHC should be able to produce the neutralino, and study its properties. Direct detection of Milky Way DM should be able to determine the DM mass and its nuclear cross section. If these are in agreement with the LHC determination of the $\tilde{\chi}_1^0$ properties, it would help confirm the important point that the Milky Way DM was indeed the neutralino. However, this would not verify explicitly that the $\tilde{\chi}_1^0$ was the DM relic particle produced during the Big Bang. To do this, one would need to see if from measurements at the LHC one could deduce the relic density as measured astronomically by WMAP [5].

In this letter we describe a series of measurements in the stau-neutralino ($\tilde{\tau}_1$-$\tilde{\chi}_1^0$) co-annihilation (CA) region. This is the region where in the early universe the stau and the neutralino annihilate at the same time to leave the correct amount of relic dark matter abundance. In this region we show how to measure the sparticle masses, confirm we are in the CA region and establish a prediction of relic density Ωh^2. Thus it will be possible to verify the fundamental connection between particle physics and cosmology. The methods introduced are general and can also be applied to other regions of the parameter space. To carry out this analysis, it is necessary to assume a model which encompasses both LHC phenomena and early universe physics. We choose here the simplest example of SUGRA (mSUGRA) [1] which depends on only one sign and four additional parameters: m_0 (universal sfermion mass), $m_{1/2}$ (universal gaugino mass), A_0 (universal soft breaking trilinear coupling constant), $\tan\beta$ ($= \langle H_1\rangle/\langle H_2\rangle$), where $\langle H_{1(2)}\rangle$ is the Higgs vacuum expectation value which gives rise to the up (down) quark masses, and the sign of μ (the bilinear Higgs coupling constant). A full specification of all these four parameters allow us to calculate Ωh^2. The procedure that we use can be generalized to more complicated models, and in fact we will see below how it is possible to test experimentally at the LHC whether gaugino universality is valid. The allowed mSUGRA parameter space with $\mu > 0$, after we include all experimental constraints, has three distinct regions picked out by the CDM constraints [6]: (i) the CA region where both m_0 and $m_{1/2}$ can be small, (ii) the focus region where the $\tilde{\chi}_1^0$ has a large Higgsino component and m_0 is very large but $m_{1/2}$ is small, and (iii) the funnel region where both m_0 and $m_{1/2}$ are large and the neutralinos can annihilate through the heavy Higgs bosons ($2M_{\tilde{\chi}_1^0} \simeq M_{A_0,H_0}$). We note that a bulk region (where none of the above properties hold) is now almost ruled out due to other experimental constraints.

We consider here the CA region with $\mu > 0$. This region is generic for a wide class of SUGRA GUT models (with or without gaugino universality). Further, if the muon $g-2$ anomaly [4] maintains, then the other two regions are essentially eliminated. The CA region has a striking characteristic in that the $\tilde{\tau}_1$ and $\tilde{\chi}_1^0$ are nearly degenerate *i.e.*, $\Delta M \equiv M_{\tilde{\tau}_1} - M_{\tilde{\chi}_1^0} \sim$ (5-15) GeV. The

TABLE I: Masses (in GeV) of SUSY particles for our reference point $m_{1/2} = 350$ GeV, $m_0 = 210$ GeV, $\tan\beta = 40$, $\mu > 0$, and $A_0 = 0$. The $\tilde{q}_L$ and $\tilde{q}_R$ masses are represented by the $\tilde{u}_L$ and $\tilde{u}_R$ masses. $\Delta M = 10.6$ GeV.

$\tilde{g}$	$\tilde{q}_L$ $\tilde{q}_R$	$\tilde{t}_2$ $\tilde{t}_1$	$\tilde{b}_2$ $\tilde{b}_1$	$\tilde{e}_L$ $\tilde{e}_R$	$\tilde{\tau}_2$ $\tilde{\tau}_1$	$\tilde{\chi}^0_2$	$\tilde{\chi}^0_1$
831	748 725	728 561	705 645	319 251	329 151.3	260.3	140.7

existence of this near degeneracy would be a strong indication that we are in the CA region. This small mass difference is characterized by a low energy tau in the $\tilde{\tau}_1 \to \tau\tilde{\chi}^0_1$ decay. It has been recently shown [7, 8] that this small mass difference can be measured at the LHC assuming A_0 and $\tan\beta$ are known (provided the τ identification is done at an efficiency of $\epsilon_\tau = 50\%$ for visible τ $p_T^{\rm vis} > 20$ GeV).

The measurements of A_0 and $\tan\beta$ cannot be avoided, since, in order to determine $\Omega_{\tilde{\chi}^0_1} h^2$, one must know all four of the mSUGRA parameters. The measurements of A_0 and $\tan\beta$, however, is difficult since the effects of these parameters are observed only in the third generation SUSY masses. It is crucial to identify the final states arising from the third generation squarks and sleptons. It is already known that the determination of stop ($\tilde{t}_1$, $\tilde{t}_2$) and sbottom ($\tilde{b}_1$, $\tilde{b}_2$) masses separately is very hard if both $\tilde{g} \to t\tilde{t}_1$ and $\tilde{g} \to b\tilde{b}_1$ can occur [9]. This is also the case in the CA region. Previously, Ref. [10] has proposed a technique to determine the four mSUGRA parameters in the bulk region under the assumption that the $\tilde{b}_1$ can be identified from the $\tilde{g} \to b\tilde{b}_1$ process along with the identification of the $\tilde{\chi}^0_2 \to \ell\ell\tilde{\chi}^0_1$ decay. Here ℓ is an electron or a muon. However, the $\tilde{g} \to t\tilde{t}_1$ decay will be a major background for the $\tilde{b}_1$ mass measurement [9]. Further, their technique cannot be applied for the case in the CA region, because the $\tilde{\chi}^0_2 \to \ell\ell\tilde{\chi}^0_1$ decay is essentially absent.

We show in this Letter, that, it is indeed possible to determine all four parameters quite accurately from measurements at the LHC once we introduce new variables involving b quarks. In particular, we directly go directly from measurements values directly to the SUSY parameters rather than the sparticle masses. This procedure is general and can be applied to other regions of the mSUGRA parameter space or to more general SUGRA models to determine the model parameters and the dark matter content. After measuring all the parameters, we then determine $\Omega_{\tilde{\chi}^0_1} h^2$ which can then be compared with the astronomical determination of $\Omega_{\rm CDM} h^2$.

We begin with a description of sparticle production and decay at the LHC. Gluinos ($\tilde{g}$) and squarks ($\tilde{q}$) are dominantly produced at the LHC, where the $\tilde{g}\tilde{q}$ production has the largest cross section. The decay chain $\tilde{q}_L \to q\tilde{\chi}^0_2 \to \tau\tilde{\tau}_1 \to \tau\tau\tilde{\chi}^0_1$ is a characteristic feature of the CA region. The branching ratios for the $\tilde{\chi}^0_2$ and $\tilde{\tau}_1$ decays are almost 100%. We analyze events in the final state of large transverse missing energy ($\not\!\!E_{\rm T}$) along with jets (j's) and τ's and show how to prove that we are indeed in the CA region. Here we consider j's that are not originated by b quarks. We then show how to measure the masses of $\tilde{g}$, $\tilde{q}_L$, $\tilde{\chi}^0_2$, $\tilde{\chi}^0_1$, and $\tilde{\tau}_1$ in a model independent way.

In order to perform our analysis, we select an mSUGRA reference point. The relevant SUSY masses are shown in Table I. We generate a sample of events using `ISAJET` [11], followed by `PGS4` [12] as a sample detector simulation. The $\not\!\!E_{\rm T}$ + j's + τ's events are selected with the following selection criteria: (a) $N_\tau \geq 2$ ($|\eta| < 2$, $p_T^{\rm vis} > 20$ GeV; but > 40 GeV for leading τ); (b) $N_j \geq 2$ ($|\eta| < 2.5$, $E_{\rm T} > 100$ GeV; > 50 GeV for other jets); (c) $\not\!\!E_{\rm T} > 180$ GeV and $E_{\rm T}{}^{j1} + E_{\rm T}{}^{j2} + \not\!\!E_{\rm T} > 600$ GeV; (d) Transverse sphericity > 0.2; (e) Veto the event if any of two leading jets is identified as b. We assume the efficiency for identifying τ jet with $p_{\rm T} > 20$ GeV is 50%, while the probability for a jet being mis-identified as τ jet is 1%. We take the b-jet tagging efficiency to be ∼42% for b jets with $E_{\rm T} > 50$ GeV and $|\eta| < 1.5$, and degrading between $1.5 < |\eta| < 2$. The tagging fake rate for c and light quarks/gluons is ∼ 9% and 2% respectively [12].

The primary SM backgrounds for this final state arise from $t\bar{t}$, W+jets and Z+jets which can be reduced by the cuts. The $p_T^{\rm vis} > 20$ GeV cut for τ's is the crucial requirement [7, 8]. In order to detect $\tilde{\chi}^0_2 \to \tau\tilde{\tau}_1 \to \tau\tau\tilde{\chi}^0_1$, we categorize a pair of taus into opposite sign (OS) and like sign (LS), and then take the OS minus LS (OS−LS) combination. This combination effectively reduces the SM events as well as combinatory backgrounds from SUSY decays.

Our analysis begins with reconstruction of the chain of the cascade decay of $\tilde{q}$s. For our sample of events, we measure the following six kinematical variables: (1) the slope, α, of the $p_T^{\rm vis}$ distribution for the lower energy τ in the OS−LS di-τ pairs, (2) the peak position $M_{\tau\tau}^{\rm peak}$ of the visible di-τ invariant mass distribution, (3) the invariant j-τ-τ mass $M_{j\tau\tau}$, (4&5) the invariant j-$\tau_{1,2}$ mass $M_{j\tau}$, (Again, none of these jets is a b jet.) and (6) We also use a scalar sum of four jets plus missing energy $M_{\rm eff} \equiv \not\!\!E_{\rm T} + \sum_{4\ {\rm jets}} E_{\rm T}{}^j$ [13], which is a function of only the $\tilde{g}$ and $\tilde{q}$ masses.

The measurement of α will tell us the existence of a low energy τ in the final state and therefore it would provide a smoking gun signal of the CA region (and hence of the SUGRA GUT model). In Fig. 1[top], we show the $p_T^{\rm vis}$ distribution obtained by the OS−LS technique. We compare it with the truth in MC and find a good agreement. In Fig. 1[bottom], we show a strong correlation of α with ΔM allowing one to determine that ΔM is small and so that we are in the CA region. It is interesting to

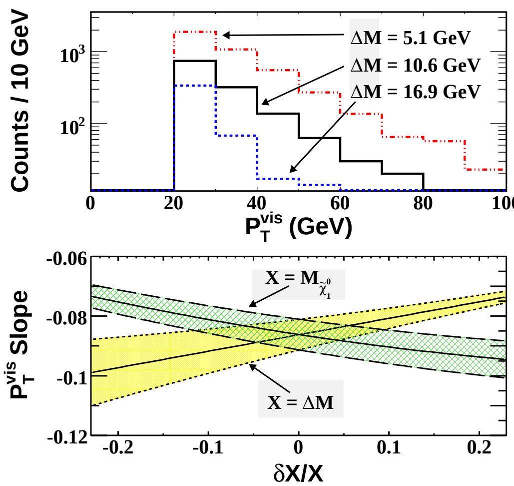

FIG. 1: [top] $p_{\rm T}^{\rm vis}$ distribution of lower energy τ's using the OS−LS technique in three samples (arbitrary luminosity) of SUSY events with $\Delta M = 5.1$, 10.6 and 16.9 GeV, where only $\tilde{\tau}_1$ masses are changed at our reference point. [bottom] the $p_{\rm T}^{\rm vis}$ slope as a function of relative change of a SUSY mass from its reference value where all other SUSY masses are fixed - two examples (ΔM and $M_{\tilde{\chi}_1^0}$) are shown with the bands corresponding to estimated uncertainties with 10 fb^{-1} of data.

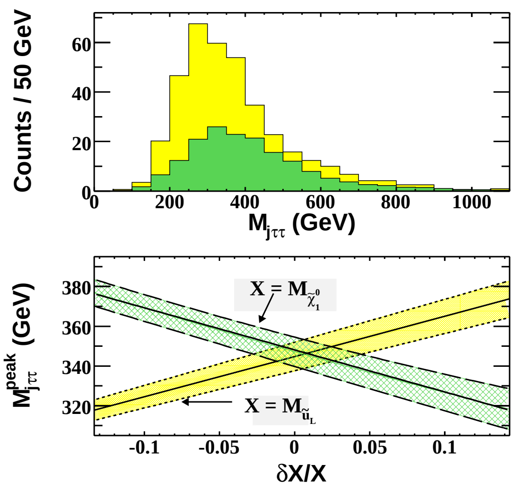

FIG. 2: [top] $M_{j\tau\tau}^{(2)}$ distributions using the OS−LS technique for SUSY events at our reference point, but with $M_{\tilde{q}_L} = 660$ GeV (yellow or light gray histogram) and 840 GeV (green or dark gray histogram), where 748 GeV at our reference point; [bottom] The peak position of the mass distribution as a function of $M_{\tilde{\chi}_1^0}$ or $M_{\tilde{q}_L}$;.

note that this observable only depends on the $\tilde{\tau}_1$ and $\tilde{\chi}_1^0$ masses. The end point of the $M_{\tau\tau}$ distribution depends upon $\tilde{\tau}_1$ and $\tilde{\chi}_{1,2}^0$ masses, which is an established technique in the study of selectron and smuon decays [13]. The end point in the stau case is challenging because of the escaping neutrinos in the τ decays. Thus a precise end point measurement requires a full understanding of the τ energy resolution and the shape of the background near the end point. This is why we choose the peak position which is less sensitive to those concerns.

The variables, $M_{j\tau\tau}$ and $M_{j\tau}$, probe $\tilde{q}_L \to q\tilde{\chi}_2^0 \to \tau\tilde{\tau}_1 \to \tau\tau\tilde{\chi}_1^0$ decay chains. To help identify these chains chains we additionally require OS−LS di-tau pairs with $M_{\tau\tau} < M_{\tau\tau}^{\rm end\ point}$ and construct $M_{j\tau\tau}$ with every jet with $E_{\rm T} > 100$ GeV in the event. If one finds three jets, there will be three masses: $M_{j\tau\tau}^{(1)}$, $M_{j\tau\tau}^{(2)}$, and $M_{j\tau\tau}^{(3)}$ in a decreasing order. Since the $\tilde{q}_L$ is lighter than the $\tilde{g}$, $M_{j\tau\tau}^{(2)}$ likely depends on the $\tilde{q}_L$, $\tilde{\chi}_2^0$ and $\tilde{\chi}_1^0$ masses for the $\tilde{g}\tilde{q}$ events. Figues 2[top] shows the $M_{j\tau\tau}^{(2)}$ distributions for two different $\tilde{q}_L$ masses and indicates that the peak value, $M_{j\tau\tau}^{(2)\ \rm peak}$, depends on the $\tilde{q}_L$ mass. In Fig. 2[bottom] we show two examples of such dependences on $M_{\tilde{q}_L}$ and $M_{\tilde{\chi}_1^0}$, keeping ΔM constant. The peak position shifts as a function of those masses. Similarly, one can show that the $M_{j\tau}^{\rm peak}$ value depends on the $\tilde{q}_L$, $\tilde{\chi}_2^0$, $\tilde{\tau}_1$ and $\tilde{\chi}_1^0$ masses. The peak position of $M_{\rm eff}$, $M_{\rm eff}^{\rm peak}$, has been shown to be a function of only the $\tilde{q}_L$ and $\tilde{g}$ masses [13].

After constructing a set of six functions as listed in Ref. [14], we invert these functions to simultaneously solve for $\tilde{g}$, $\tilde{\chi}_{1,2}^0$, $\tilde{\tau}_1$, and average $\tilde{q}_L$ masses and their uncertainties. At 10 fb^{-1} we obtain the masses (in GeV): $M_{\tilde{g}} = 831 \pm 28$, $M_{\tilde{\chi}_2^0} = 260 \pm 15$, $M_{\tilde{\chi}_1^0} = 141 \pm 19$, $\Delta M = 10.6 \pm 2.0$. and $M_{\tilde{q}_L} = 748 \pm 25$. We can also check the universality of gaugino masses at the GUT scale which implies at the electroweak scale $r_1 = M_{\tilde{g}}/M_{\tilde{\chi}_1^0} = 5.91$ and $r_2 = M_{\tilde{g}}/M_{\tilde{\chi}_2^0} = 3.19$. For 10 fb^{-1}, we find that $r_1 = 5.90 \pm 0.83$ and $r_2 = 3.12 \pm 0.15$, testing the universality relations to 14% and 5%, respectively.

Our primary focus is to calculate $\Omega_{\tilde{\chi}_1^0} h^2$. To do this we determine the mSUGRA parameters. Since $M_{\rm eff}$ and $M_{j\tau\tau}$ depend only on $\tilde{q}_L$ (first two generations), $\tilde{g}$, $\tilde{\chi}_2^0$ and $\tilde{\chi}_1^0$ masses, we can determine m_0 and $m_{1/2}$ from these two variables. In Fig. 3 we show the dependences of the peak positions of these variables on m_0 and $m_{1/2}$. We can use these peak positions to determine m_0 and $m_{1/2}$ directly without requiring any knowledge of A_0 or $\tan\beta$.

In order to determine A_0 and $\tan\beta$, we use two variables which depend on A_0 and $\tan\beta$, equivalently, on $\tilde{\tau}_1$, $\tilde{t}_1$ and $\tilde{b}_1$ masses. The $M_{\tau\tau}^{\rm peak}$ depends on $\tilde{\tau}_1$ mass and a new variable $M_{\rm eff}^{(b)}$ depends on $\tilde{t}_1$ and $\tilde{b}_1$ masses. The $M_{\rm eff}^{(b)}$ variable is similar to $M_{\rm eff}$, but the leading jet of the four jets is required to be a b jet. Since $\tilde{t}_1$ and $\tilde{b}_1$ decays always produce a b jet in the final state, it can be expressed as a function of $\tilde{t}_1$ and $\tilde{b}_1$ masses. We plot $M_{\tau\tau}^{\rm peak}$ $M_{\rm eff}^{(b)\ \rm peak}$ as functions of A_0 and $\tan\beta$ in Fig. 4.

Now combining these four measurements we determine m_0, $m_{1/2}$, A_0 and $\tan\beta$ with remarkably low uncertainties: $m_0 = 205 \pm 4$ GeV; $m_{1/2} = 350 \pm 4.2$ GeV; $A_0 = 0 \pm 16$ GeV; $\tan\beta = 40 \pm 0.8$ for a luminosity of 10 fb^{-1}. The above errors are statistical only. This process of de-

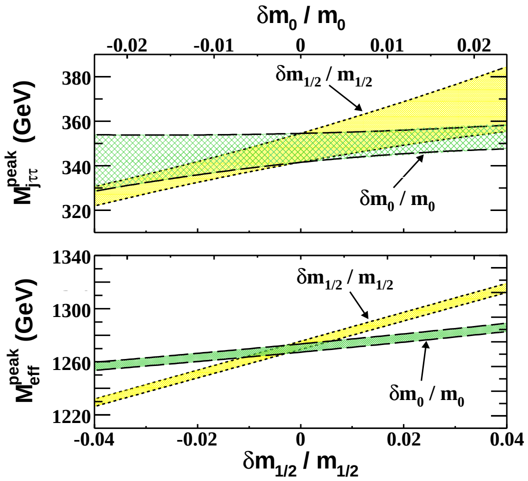

FIG. 3: Dependence of $M_{j\tau\tau}^{(2)\ \mathrm{peak}}$ (top) and $M_{\mathrm{eff}}^{\mathrm{peak}}$ (bottom) as a function of $m_{1/2}$ and m_0.

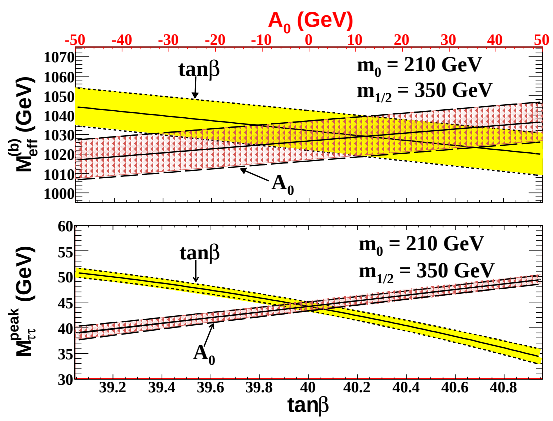

FIG. 4: Dependence of $M_{\tau\tau}^{\mathrm{peak}}$ (top) and $M_{\mathrm{eff}}^{(b)\ \mathrm{peak}}$ (bottom) as a function of $\tan\beta$ and A_0.

termining the mSUGRA parameters is very general and can be applied to other SUGRA models. In the determination of the mSUGRA parameters, we can also use other variables, *e.g.*, $M_{b\tau_{1,2}}$,$M_{b\tau\tau}$ etc. to further improve the accuracy.

After we have measured all four mSUGRA variables, we calculate $\Omega_{\tilde{\chi}_1^0}h^2$ using DARKSUSY [15]. The sign of μ is chosen to be > 0 as preferred two measurements of the $b \to s\gamma$ decay branching ratio and the muon $g-2$. In the CA region, the $\Omega_{\tilde{\chi}_1^0}h^2$ calculation depends on crucially ΔM due to the Boltzmann suppression factor $\exp[\Delta M/kT]$ in the relic density formula [16]. In Fig. 5, we plot the 1σ uncertainty ellipse in Ωh^2-ΔM plane and find that the uncertainty is 11 (4.8)% at 10 (50) fb^{-1}. It is quite interesting to note that the uncertainty at 30 fb^{-1} is 6.2%, comparable to that of the WMAP measurement [5].

In conclusion, we have established a technique to make a precision measurement of the relic density of the $\tilde{\chi}_1^0$ in the $\tilde{\tau}_1$-$\tilde{\chi}_1^0$ CA region of the mSUGRA model at the

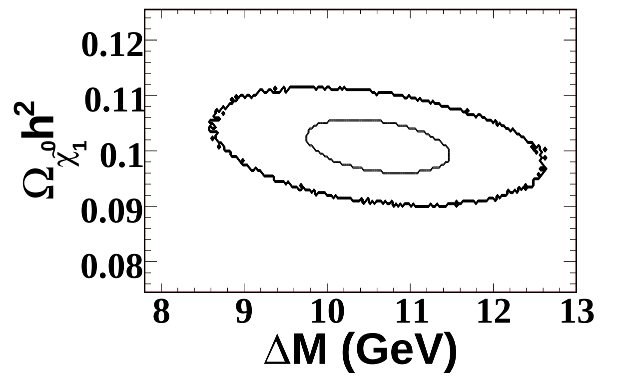

FIG. 5: Contour plot of 1σ uncertainty in Ωh^2-ΔM plane for 10 fb^{-1} (outer ellipse) and 50 fb^{-1} (inner ellipse) at the LHC.

LHC. This is done using only mSUGRA parameters, determined by the kinematical analyses of a sample of $\not{E}_T$ + j's + τ's events and the scalar sum of $\not{E}_T$ + four jets (with and without b jets). The accuracy of the relic density calculation at 30 fb^{-1} is expected to be comparable to the accuracy in the WMAP measurement. Therefore, it is possible to confirm that with measurements purely at the LHC, the SUSY origin of dark matter was created in the early universe.

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