Extrapolation Technique Pitfalls in Asymmetry Measurements at Colliders

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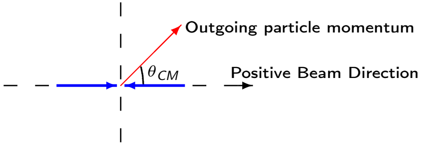
Overview

Background, Motivation, and Goals

- It is common at colliders to measure asymmetries
  - e.g. Forward-Backward top quark pair production at the Tevatron
- Measurements in disagreement with SM NLO predictions are exciting – so we need to be confident in these measurements
- Often data is only measured in a visible region of the detector, A^{visible}
- This necessitates a method to measure the inclusive physical value, $A^{\text{inclusive}}$



- A simple yet powerful method often applicable is the use of a constant extrapolation factor

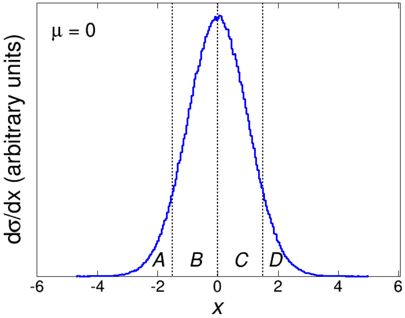
$$A^{\text{inclusive}} = R \cdot A^{\text{visible}}$$

Background, Motivation, and Goals

- Monte Carlo (MC) simulation (often a single pseudo-experiment) is typically used to estimate R
- So we want to understand two things through this study:
  - whether we can confidently and reliably use a constant R to perform the extrapolation, and
  - what the required MC sample size is to be able to reliably estimate R

Monte Carlo Study

- In this study we use a simple single Gaussian model[1] with a mean, μ ($\mu \propto A$)
- A pseudo-experiment (PE) refers to generating a Gaussian distribution with a specified number of events, N



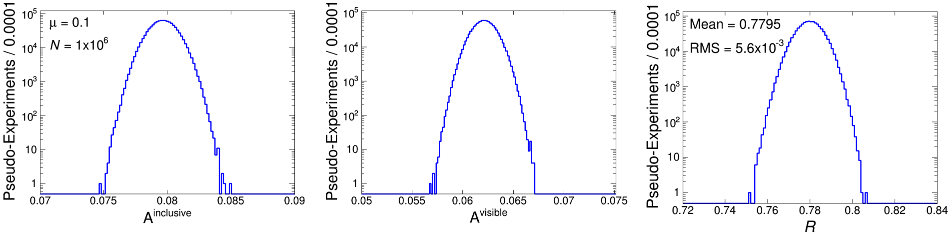
$$A^{inclusive} = \frac{(C+D)-(A+B)}{A+B+C+D}$$

$$A^{visible} = \frac{C-B}{B+C}$$

- $\mu = 0.1$ corresponds to $A^{\text{inclusive}} \approx 8\%$ which is typically seen in A_{FB} $t\bar{t}$ measurements

[1] Z. Hong, R. Edgar, S. Henry, D. Toback, J. S. Wilson, and D. Amidei, Phys. Rev. D 90 (2014) 014040.

- Measure $A^{\text{inclusive}}$, A^{visible}, and R from a single PE
- With many PEs, we get distributions for $A^{\text{inclusive}}$, A^{visible}, and R:

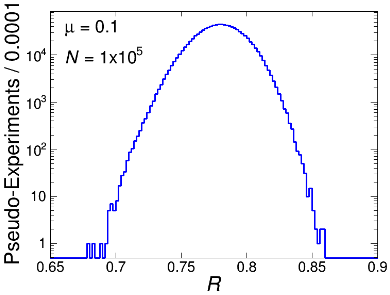


- What happens to the R distribution as we vary N and μ?

Monte Carlo Study: Fix μ and Vary N

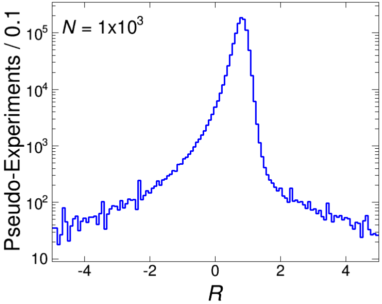
Fixed Value of μ with high statistics:

Reliable measurement of R



Fixed Value of μ with low statistics:

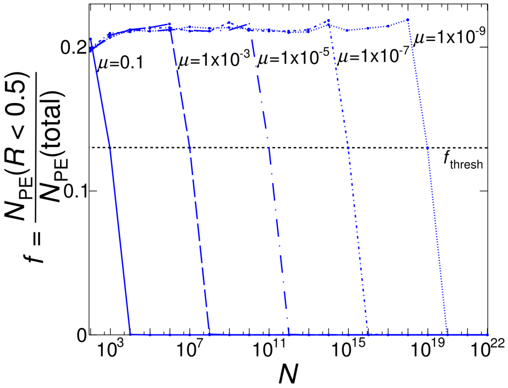
Unreliable measurement of R



- With large enough sample size, measurements of R are <u>very accurate</u>
- As N decreases, measurement of R becomes <u>unreliable</u>, and can no longer correctly reproduce $A^{\text{inclusive}}$ from A^{visible}
- This transition is observed for all values of μ

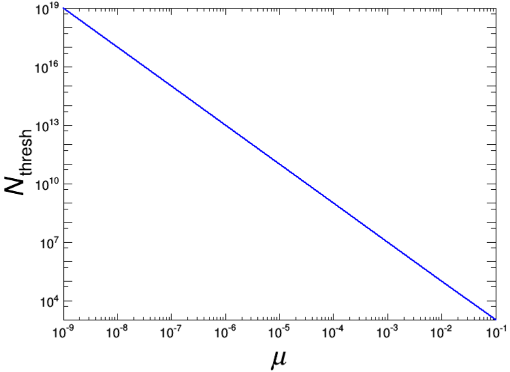
Monte Carlo Study: Quantifying the Transition for Varying μ

- We now examine how many events, N_{thresh}, are needed to give reliable measurements of R



- We define f as the fraction of pseudo-experiments with $R < 0.5$
- This *should be* many σ from the mean, so we require $f \approx 0$
- To examine/quantify the behavior for reliable measurements, we define a threshold value, f_{thresh}, and examine the relationship between N_{thresh} and μ

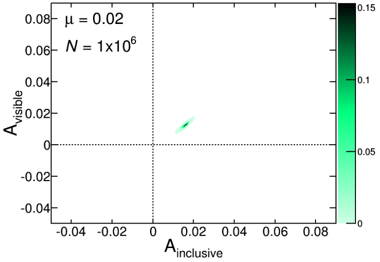
Monte Carlo Study: Results



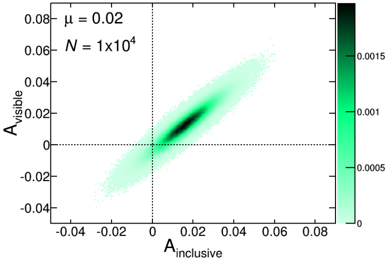
- We find that it can take a much-larger-than-expected sample size to reliably measure R, especially for very small μ (or equivalently A)
  - N_{thresh} rises as $\frac{1}{\mu^2}$ or $\frac{1}{A^2}$
- We also find that when N is large enough for reliable measurements, R is measured to be close to constant for all values of μ

Closed Form Statistical Validation: Number of Events Required for Reliable Measurements

"Enough" Events in Simulation – Reliable Measurements



"Not Enough" Events in Simulation – Unreliable Measurements



- Require $A^{inclusive}$ (denominator of R) to be greater than *at least* 1σ away from 0

Closed Form Statistical Validation: Number of Events Required for Reliable Measurements

- We can use statistics to answer the question of how many events, N_{thresh}, are required for this condition to be satisifed

$$A^{\text{inclusive}} \geq \sigma_{A_{FB}{}^{\text{inclusive}}}$$

- We are able to find N_{thresh} as a function of μ for our single Gaussian model (calculation in backup slides):

$$N_{\text{thresh}} \geq 2 \cdot \frac{\left(1 + \operatorname{erf}\left(\frac{\mu}{\sqrt{2}}\right)\right)}{\operatorname{erf}\left(\frac{\mu}{\sqrt{2}}\right)^2}$$

- Some limiting cases:
  - As $\mu \to 0$, $N_{\text{thresh}} \to \infty$
  - $\operatorname{erf}\left(\frac{\mu}{\sqrt{2}}\right) \approx \sqrt{\frac{2}{\pi}}\mu$ for small μ, so we find that $N_{\text{thresh}} \propto \frac{1}{\mu^2}$ which is precisely what we just saw in the MC study

Closed Form Numerical Validation: Is R constant?

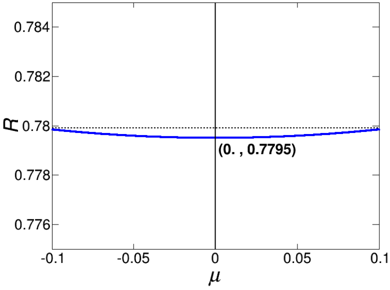
- Examining the behavior of R as a function of μ analytically is straightforward for the single Gaussian model
  - We set $\sigma = 1.0$ and use the visible region $|x| < 1.5$
- For large values of μ (i.e 0.1), R only rises by 0.04% relative to $R(\mu = 0)$

Recall:

$$A^{inclusive} = \frac{(C + D) - (A + B)}{A + B + C + D}$$

$$A^{visible} = \frac{C - B}{B + C}$$

$$R = \frac{A^{visible}}{A^{inclusive}}$$



Conclusions

- We have studied the multiplicative extrapolation of A^{visible} to $A^{\text{inclusive}}$ for the single Gaussian model, and while a custom study would be needed for any non-Gaussian physics distribution, we have observed that a linear extrapolation can be used in this and other similar cases

- While MC methods work reliably (even for small A), they can require much larger sample sizes than expected, rising as $\frac{1}{A^2}$

- Our results have the potential to be applied to many different asymmetry measurements in collider experiments, and have already been useful at the Tevatron for the $t\bar{t}$ forward-backward asymmetry

Backups: The Cauchy Distribution

- A distribution of the ratio of two independent Gaussian variables
- Mean and RMS are undefined; though mode and median are well defined
- $A^{\text{inclusive}}$ and A^{visible} are approximately Gaussian, thus as the mean of the $A^{\text{inclusive}}$ distribution approaches 0, R begins approximating a Cauchy distribution

Backups: The Statistical Solution Calculation

We need enough statistics such that $A_{FB}^{inclusive}$, the denominator of R, is more than 1 sigma away from 0 (we will set it to be k, where k will be determined later). In other words, we want to know how many events it takes in a pseudo-experiment to ensure the mean of the full asymmetry will be k standard-deviations away from zero.
To do this we start with the equation

$$\sigma_{A_{FB}^{inclusive}} = \frac{A_{FB}^{inclusive}}{k} \tag{1}$$

where $\sigma_{AFB}^{inclusive}$ is the variation (or uncertainty) of the measured value of $A_{FB}^{inclusive}$. We will find both $\sigma_{A_{FB}^{inclusive}}$ and $A_{FB}^{inclusive}$ as functions of N and μ and substitute them into Eq. 1 to get the functional relation between N and μ for "good statistics".

Backups: The Statistical Solution Calculation

We begin with our definition of asymmetry,

$$A_{FB}^{inclusive} = \frac{N_+ - N_-}{N_+ + N_-} \tag{2}$$

where $N_+ = C + D$ and $N_- = A + B$ as on Slide 2. Next we define $N = N_+ + N_-$ as the total number of events in the original Gaussian distribution, and rewrite this as:

$$A_{FB}^{inclusive} = \frac{2N_+ - N}{N}. \tag{3}$$

We note that since our distributions are Gaussian, we can write N_+ in terms of N and μ, with the relation given by

$$\begin{aligned} N_+ &= \frac{N}{\sqrt{2\pi}} \int_0^\infty \mathrm{dx}\ e^{-(x-\mu)^2/2} \\ &= \frac{N}{2}\left(\mathrm{erf}\left(\frac{\mu}{\sqrt{2}}\right) + 1\right) \end{aligned} \tag{4}$$

Plugging this in to Eq. 3 and reducing, we get

$$A_{FB}^{inclusive} = \frac{\cancel{2}\frac{\cancel{N}}{\cancel{2}}\left(\text{erf}\left(\frac{\mu}{\sqrt{2}}\right) + \cancel{1}\right) - \cancel{N}}{\cancel{N}}$$
$$= \text{erf}\left(\frac{\mu}{\sqrt{2}}\right) \tag{5}$$

We next find $\sigma_{A_{FB}^{inclusive}}$ by beginning with the definition given in Bevington (92) applied to our problem,

$$\sigma_{A_{FB}^{inclusive}} = \left(\frac{\partial A_{FB}^{inclusive}}{\partial N_+}\right)\sigma_{N_+} + \left(\frac{\partial A_{FB}^{inclusive}}{\partial N}\right)\sigma_N. \tag{6}$$

Taking a simple derivative of $A_{FB}^{inclusive}$ from Eq. 3 gives us

$$\left(\frac{\partial A_{FB}^{inclusive}}{\partial N_+}\right) = \frac{2}{N} \tag{7}$$

Backups: The Statistical Solution Calculation

To be consistent with the previous study, we fix N and allow N_+ to vary. This means that $\sigma_N = 0$, and from simple statistics

$$\sigma_{N_+} = \sqrt{N_+} \tag{8}$$

Plugging Eqs. 7 and 8 into Eq. 6, we get

$$\sigma_{A_{FB}^{inclusive}} = \frac{2}{N} \cdot \sqrt{N_+}. \tag{9}$$

Plugging Eq. 4 into this, we get

$$\begin{aligned} \sigma_{A_{FB}^{inclusive}} &= \frac{2}{N} \cdot \sqrt{\frac{N}{2}\left(\mathrm{erf}\left(\frac{\mu}{\sqrt{2}}\right) + 1\right)} \\ &= \sqrt{\frac{2}{N}} \cdot \sqrt{\left(1 + \mathrm{erf}\left(\frac{\mu}{\sqrt{2}}\right)\right)} \end{aligned} \tag{10}$$

Backups: The Statistical Solution Calculation

Finally, plugging Eqs. 5 and 10 back into Eq. 1 gives us

$$\sqrt{\frac{2}{N}} \cdot \sqrt{\left(1 + \operatorname{erf}\left(\frac{\mu}{\sqrt{2}}\right)\right)} = \frac{\operatorname{erf}(\frac{\mu}{\sqrt{2}})}{k}, \tag{11}$$

and solving for N, we get

$$N = \frac{2k^2\left(1 + \operatorname{erf}\left(\frac{\mu}{\sqrt{2}}\right)\right)}{\operatorname{erf}\left(\frac{\mu}{\sqrt{2}}\right)^2} \tag{12}$$

This is, as we set out to solve for, the number of events it takes per pseudo-experiment to ensure the mean of the full asymmetry will be k standard-deviations away from zero, and thus give good statistics. Discussion of the implication of this result is included in the main slides.

Backups: Closed Form Numerical Solution Gaussian Functions

$$A_{FB}^{inclusive} = \frac{\frac{1}{\sqrt{2\pi}\sigma}\int_0^{\infty} dx\Big[\exp(-\frac{(x-\mu)^2}{2\sigma^2}) - \exp(-\frac{(-x-\mu)^2}{2\sigma^2})\Big]}{\frac{1}{\sqrt{2\pi}\sigma}\int_0^{\infty} dx\Big[\exp(-\frac{(x-\mu)^2}{2\sigma^2}) + \exp(-\frac{(-x-\mu)^2}{2\sigma^2})\Big]}$$

$$A_{FB}^{visible} = \frac{\frac{1}{\sqrt{2\pi}\sigma}\int_0^{1.5} dx\Big[\exp(-\frac{(x-\mu)^2}{2\sigma^2}) - \exp(-\frac{(-x-\mu)^2}{2\sigma^2})\Big]}{\frac{1}{\sqrt{2\pi}\sigma}\int_0^{1.5} dx\Big[\exp(-\frac{(x-\mu)^2}{2\sigma^2}) + \exp(-\frac{(-x-\mu)^2}{2\sigma^2})\Big]}$$

$$R = \frac{A_{FB}^{visible}}{A_{FB}^{inclusive}}$$