Alberta Thy 11-00
BNL-HET-00/35
October 2000
hep-ph/0010194

The Muon Anomalous Magnetic Moment: Standard Model Theory and Beyond

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QED, Hadronic, and Electroweak Standard Model contributions to the muon anomalous magnetic moment, $a_\mu \equiv (g_\mu - 2)/2$, are reviewed. Theoretical uncertainties in the prediction $a_\mu^{\rm SM} = 116\,591\,597(67) \times 10^{-11}$ are scrutinized. Effects due to "New Physics" are described. Implications of the current experiment vs. theory constraint $a_\mu^{\rm exp} - a_\mu^{\rm SM} = 453(465) \times 10^{-11}$ and anticipated near term error reduction to $\pm 155 \times 10^{-11}$ are discussed.

Presented at the

5th International Symposium on Radiative Corrections
(RADCOR–2000)
Carmel CA, USA, 11–15 September, 2000

1 Introduction

Leptonic anomalous magnetic moments provide precision tests of the Standard Model and stringent constraints on potential "New Physics" effects. In the case of the electron, comparing the extraordinary measurements of $a_e \equiv (g_e - 2)/2$ at the University of Washington [1]

$$\begin{aligned} a_{e^-}^{\text{exp}} &= 0.001\,159\,652\,188\,4(43), \\ a_{e^+}^{\text{exp}} &= 0.001\,159\,652\,187\,9(43), \end{aligned} \tag{1}$$

with the prediction [2,3,4,5]

$$\begin{aligned} a_e^{\text{SM}} = & \frac{\alpha}{2\pi} - 0.328\,478\,444\,00 \left(\frac{\alpha}{\pi}\right)^2 + 1.181\,234\,017 \left(\frac{\alpha}{\pi}\right)^3 \\ & -1.5098(384) \left(\frac{\alpha}{\pi}\right)^4 + 1.66(3) \times 10^{-12} (\text{hadronic \& electroweak loops}) \end{aligned} \tag{2}$$

provides the best determination of the fine structure constant [6],

$$\alpha^{-1}(a_e) = 137.035\,999\,58(52). \tag{3}$$

To test the Standard Model requires an alternative measurement of α with comparable accuracy. Unfortunately, the next best determination of α, from the quantum Hall effect [2],

$$\alpha^{-1}(qH) = 137.036\,003\,00(270), \tag{4}$$

has a much larger error. If one assumes that $\left|\Delta a_e^{\text{New Physics}}\right| \simeq m_e^2/\Lambda^2$, where Λ is the scale of "New Physics", then the agreement between $\alpha^{-1}(a_e)$ and $\alpha^{-1}(qH)$ currently probes $\Lambda \lesssim \mathcal{O}\,(100 \text{ GeV})$. To access the much more interesting $\Lambda \sim \mathcal{O}\,(\text{TeV})$ region would require an order of magnitude improvement in a_e^{exp} (technically feasible [7]), an improved calculation of the 4-loop QED contribution to a_e^{SM} and a much better independent measurement of α^{-1} by almost two orders of magnitude. The last requirement, although difficult, is perhaps most likely to come [6] from combining the already precisely measured Rydberg constant with a much better determination of m_e.

We should note that for "New Physics" effects that are linear in the electron mass, $\Delta a_e^{\text{NP}} \sim m_e/\Lambda$, naively, one is currently probing a much more impressive $\Lambda \sim \mathcal{O}\,(10^7 \text{ GeV})$ and the possible advances described above would explore $\mathcal{O}\,(10^9 \text{ GeV})$! However, we subsequently argue that such linear "New Physics" effects are misleading or unlikely.

The measurement of the muon's anomalous magnetic moment has also been impressive. A series of experiments at CERN that ended in 1977 found [8]

$$a_\mu^{\text{exp}} = 116\,592\,300(840) \times 10^{-11} \qquad (\text{CERN 1977}). \tag{5}$$

More recently, an ongoing experiment (E821) at Brookhaven National Laboratory has been running with much higher statistics and a very stable, well measured magnetic field in its storage ring. Based on data taken through 1998, combined with the earlier CERN result in (5), it found [9]

$$a_\mu^{\rm exp} = 116\,592\,050(460) \times 10^{-11} \qquad \text{(CERN'77+BNL'98)}. \tag{6}$$

Ongoing analysis of E821's 1999 data is expected to reduce the error in (6) to about $\pm 140 \times 10^{-11}$ before the end of this year (2000). The ultimate goal of the experiment is $\pm 40 \times 10^{-11}$, about a factor of 20 improvement relative to the classic CERN experiments.

Although $a_\mu^{\rm exp}$ is currently about 1000 times less precise than $a_e^{\rm exp}$, it is much more sensitive to hadronic and electroweak quantum loops as well as "New Physics" effects, since such contributions [10] are generally proportional to m_l^2. The $m_\mu^2/m_e^2 \simeq 40\,000$ enhancement more than compensates for the reduced experimental precision and makes $a_\mu^{\rm exp}$ a more sensitive probe of short-distance phenomena. Indeed, as we later illustrate, a deviation in $a_\mu^{\rm exp}$ from the Standard Model prediction, $a_\mu^{\rm SM}$, could quite naturally be interpreted as the appearance of "New Physics" such as supersymmetry, an exciting prospect. Of course, before making such an interpretation, one must have a reliable prediction for $a_\mu^{\rm SM}$, an issue that we address in the next section.

Before leaving the comparison between $a_e^{\rm exp}$ and $a_\mu^{\rm exp}$, we should remark that for cases where "New Physics" contributions to a_l scale as m_l/Λ, roughly equal sensitivity in Λ ($\sim 10^7$ GeV) currently exists for both types of measurements. However, as previously mentioned, such examples are in our view artificial.

2 Standard Model Prediction For a_μ

2.1 QED Contribution

The QED contribution to a_μ has been computed through 5 loops [5,2]

$$\begin{aligned} a_\mu^{\rm QED} &= \frac{\alpha}{2\pi} + 0.765\,857\,376(27)\left(\frac{\alpha}{\pi}\right)^2 + 24.050\,508\,98(44)\left(\frac{\alpha}{\pi}\right)^3 \\ &\quad + 126.07(41)\left(\frac{\alpha}{\pi}\right)^4 + 930(170)\left(\frac{\alpha}{\pi}\right)^5 . \end{aligned} \tag{7}$$

Growing coefficients in the α/π expansion reflect the presence of large $\ln m_\mu/m_e \simeq 5.3$ terms coming from electron loops. Employing the value of α from a_e in eq. (3) leads to

$$a_\mu^{\rm QED} = 116\,584\,705.7(2.9) \times 10^{-11}. \tag{8}$$

The current uncertainty is well below the $\pm 40 \times 10^{-11}$ ultimate experimental error anticipated from E821 and should, therefore, play no essential role in the confrontation between theory and experiment.

2.2 Hadronic Loop Corrections

Starting at $\mathcal{O}(\alpha^2)$, hadronic loop effects contribute to a_μ via vacuum polarization. A first principles QCD calculation of that effect does not exist. Fortunately, it is possible to evaluate the leading effect via the dispersion integral [11]

$$a_\mu^{\rm Had}(\text{vac. pol.}) = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} \mathrm{d}s\, K(s)\ \sigma^0(s)_{e^+e^-\to\text{hadrons}}, \tag{9}$$

where $\sigma^0(s)_{e^+e^-\to\text{hadrons}}$ means QED vacuum polarization and other extraneous radiative corrections have been subtracted from measured cross sections, and

$$\begin{aligned} K(s) &= x^2\left(1-\frac{x^2}{2}\right) + (1+x)^2\left(1+\frac{1}{x^2}\right)\left[\ln(1+x) - x + \frac{x^2}{2}\right] + \frac{1+x}{1-x}\,x^2\ln x \\ x &= \frac{1-\sqrt{1-4m_\mu^2/s}}{1+\sqrt{1-4m_\mu^2/s}}. \end{aligned} \tag{10}$$

Detailed studies of eq. (9) have been carried out by a number of authors [12,13,14,15,16,17,18]. Here, we employ an analysis due to Davier and Höcker [13,14,15] which finds

$$a_\mu^{\rm Had}(\text{vac. pol.}) = 6924(62)\times 10^{-11}. \tag{11}$$

It used experimental e^+e^- data, hadronic tau decays, perturbative QCD and sum rules to minimize the uncertainty in that result. The contributions coming from various energy regions are illustrated in Table 1.

Table 1: Contributions to $a_\mu^{\rm Had}$(vac. pol.) from different energy regions as found by Davier and Höcker [13,14,15].

$\sqrt{s}$ (GeV)	$a_\mu^{\rm Had}(\text{vac. pol.})\times 10^{11}$
$2m_\pi - 1.8$	6343 ± 60
$1.8-3.7$	338.7 ± 4.6
$3.7-5+\psi(1S,2S)$	143.1 ± 5.4
$5-9.3$	68.7 ± 1.1
$9.3-12$	12.1 ± 0.5
$12-\infty$	18.0 ± 0.1
Total	6924 ± 62

It is clear from Table 1 that the final result and its uncertainty are dominated by the low energy region. In fact, the $\rho(770\text{ MeV})$ resonance provides about 72% of the total hadronic contribution to $a_\mu^{\rm Had}$(vac. pol.).

To reduce the uncertainty in the ρ resonance region, Davier and Höcker employed $\Gamma(\tau \to \nu_\tau \pi^- \pi^0)/\Gamma(\tau \to \nu_\tau \bar{\nu}_e e^-)$ data to supplement $e^+e^- \to \pi^+\pi^-$ cross-sections. In the $I = 1$ channel they are related by isospin. Currently, tau decay data is experimentally more precise.

An issue in the use of tau decay data is the magnitude of isospin violating corrections due to QED and the $m_d - m_u$ mass difference. A short-distance QED correction [19] of about -2% was applied to the hadronic tau decay data and the $m_{\pi^\pm} - m_{\pi^0}$ phase space difference is easy to account for. Other isospin violating differences are estimated to be about $\pm 0.5\%$ and included in the hadronic uncertainty.

Although the error assigned to the use of tau decay data appears reasonable, it has been questioned [20,21]. More recent preliminary $e^+e^- \to \pi^+\pi^-$ data from Novosibirsk [20] seems to suggest a potential 1.5 sigma difference with corrected hadronic tau decays which would seem to further reduce $a_\mu^{\rm Had}$. It is not clear whether the difference is due to additional isospin violating corrections to hadronic tau decays or radiative corrections to $e^+e^- \to$ hadrons data which must be accounted for in any precise comparison [22]. If that difference is confirmed by further scrutiny, it could lead to a reduction in $a_\mu^{\rm Had}$(vac. pol.). Resolution of this issue is extremely important. However, we note that a reduction in $a_\mu^{\rm Had}$ would further increase the $a_\mu^{\rm exp} - a_\mu^{\rm SM}$ difference given in the abstract which is roughly 1 sigma at present.

Evaluation of the 3-loop hadronic vacuum polarization contribution to a_μ has been updated to [23,17]

$$\Delta a_\mu^{\rm Had}(\text{vac. pol.}) = -100(6) \times 10^{-11}. \tag{12}$$

Light-by-light hadronic diagrams have been evaluated using chiral perturbation theory. An average [13,14,15] of two recent studies [24,25] gives

$$\Delta a_\mu^{\rm Had}(\text{light-by-light}) = -85(25) \times 10^{-11}. \tag{13}$$

Adding the contributions in Eqs. (11), (12), and (13) leads to the total hadronic contribution

$$a_\mu^{\rm Had} = 6739(67) \times 10^{-11}. \tag{14}$$

The uncertainty in that result represents the main theoretical error in $a_\mu^{\rm SM}$. It would be very valuable to supplement the above evaluation of $a_\mu^{\rm Had}$ with lattice calculations (for the light-by-light contribution) and improved e^+e^- data. A goal of $\pm 40 \times 10^{-11}$ or smaller appears to be within reach and is well matched to the prospectus of experiment E821 at Brookhaven which aims for a similar level of accuracy.

2.3 Electroweak corrections

The one-loop electroweak radiative corrections to a_μ are predicted in the Standard Model to be [26,27,28,29,30,31,32]

$$\begin{aligned} a_\mu^{\rm EW}(1\,\text{loop}) &= \frac{5}{3}\frac{\mathrm{G}_\mu \mathrm{m}_\mu^2}{8\sqrt{2}\pi^2} \\ &\times \left[1+\frac{1}{5}(1-4\sin^2\theta_W)^2 + \mathcal{O}\left(\frac{m_\mu^2}{M^2}\right)\right] \\ &\approx 195\times 10^{-11} \end{aligned} \tag{15}$$

where $G_\mu = 1.16637(1)\times 10^{-5}$ GeV^{-2}, $\sin^2\theta_W \equiv 1 - M_W^2/M_Z^2 \simeq 0.223$. and $M = M_W$ or $M_{\rm Higgs}$. The original goal of E821 at Brookhaven was to measure that predicted effect at about the 5 sigma level (assuming further reduction in the hadronic uncertainty). Subsequently, it was pointed out [33] that two-loop electroweak contributions are relatively large due to the presence of $\ln m_Z^2/m_\mu^2 \simeq 13.5$ terms. A full two-loop calculation [34,35], including low-energy hadronic electroweak loops [36,35], found for $m_H \simeq 150$ GeV

$$a_\mu^{\rm EW}(2\text{ loop}) = -43(4)\times 10^{-11}, \tag{16}$$

where the quoted error is a conservative estimate of hadronic, Higgs, and higher-order corrections. Combining eqs. (15) and (16) gives the electroweak contribution

$$a_\mu^{\rm EW} = 152(4)\times 10^{-11}. \tag{17}$$

Higher-order leading logs of the form $(\alpha \ln m_Z^2/m_\mu^2)^n$, $n = 2, 3, \ldots$ can be computed via renormalization group techniques [37]. Due to cancellations, they give a relatively small $+0.5\times 10^{-11}$ contribution to $a_\mu^{\rm EW}$. It is safely included in the uncertainty of eq. (17).

2.4 Comparison with Experiment

The complete Standard Model prediction for a_μ is

$$a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm Had} + a_\mu^{\rm EW}. \tag{18}$$

Combining eqs. (8), (14) and (17), one finds

$$a_\mu^{\rm SM} = 116\,591\,597(67)\times 10^{-11}. \tag{19}$$

Comparing that prediction with the current experimental value in eq. (6) gives

$$a_\mu^{\rm exp} - a_\mu^{\rm SM} = 453 \pm 465\times 10^{-11}. \tag{20}$$

That still leaves considerable room for contributions from "New Physics" beyond the Standard Model. At (one-sided [38]) 95% CL, one finds

$$-310 \times 10^{-11} \leq a_\mu(\text{New Physics}) \leq 1216 \times 10^{-11}. \tag{21}$$

That constraint is already significant for theories which give additional negative contributions to a_μ. Soon, its range will be reduced by a factor of 3 when the new E821 result is unveiled. Will a clear signal for "New Physics" emerge? As we show in the next section, realistic examples of "New Physics" could quite easily lead to $a_\mu(\text{New Physics}) \sim \mathcal{O}(400 - 500 \times 10^{-11})$ which would appear as about a 3 sigma effect in the near term and increase to a 6 or 7 sigma effect as E821 is completed and the hadronic uncertainties in $a_\mu^{\rm SM}$ are further reduced.

3 "New Physics" effects

In general, "New Physics" (i.e. beyond the Standard Model expectations) will contribute to a_μ via quantum loop effects. Indeed, whenever a new model or Standard Model extension is proposed, $a_\mu^{\rm exp} - a_\mu^{\rm SM}$ is employed to constrain or rule it out. Future improvements in $a_\mu^{\rm exp}$ will make such tests even more powerful. Alternatively, they may in fact uncover a significant deviation indicative of "New Physics".

In this section we describe several generic examples of interesting "New Physics" probed by $a_\mu^{\rm exp} - a_\mu^{\rm SM}$. Rather than attempting to be inclusive, we concentrate on two general scenarios: 1) Supersymmetric loop effects which can be substantial and would be heralded as the most likely explanation if a deviation in $a_\mu^{\rm exp}$ is observed and 2) Models of radiative muon mass generation which predict $a_\mu(\text{New Physics}) \sim m_\mu^2/M^2$ where M is the scale of "New Physics". Other examples of potential "New Physics" contributions to a_μ are only briefly mentioned.

3.1 Supersymmetry

The supersymmetric contributions to a_μ stem from smuon–neutralino and sneutrino-chargino loops (see Fig. 1). They include 2 chargino and 4 neutralino states and could in principle entail slepton mixing and phases.

Early studies of the supersymmetric contributions $a_\mu^{\rm SUSY}$ were carried out in the context of the minimal SUSY standard model (MSSM) [39,40,41,42,43,44,45,46], in an E_6 string-inspired model [47,48], and in an extension of the MSSM with an additional singlet [49,50]. An important observation was made in [51], namely that some of the contributions are enhanced by the ratio of Higgs' vacuum expectation values, $\tan\beta$, which in some models is large (of order $m_t/m_b \approx 40$). The main contribution is generally due to the chargino-sneutrino diagram (Fig. 1a), which is enhanced by a Yukawa coupling in the muon-sneutrino-Higgsino vertex.

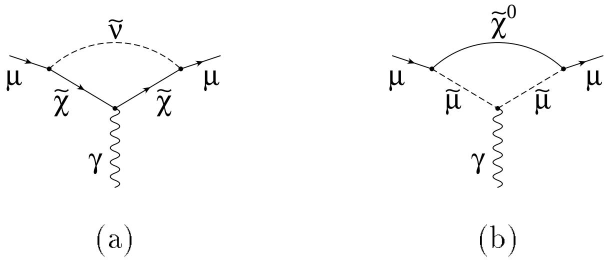

Figure 1: Supersymmetric loops contributing to the muon anomalous magnetic moment.

The leading effect is approximately given in the large $\tan\beta$ limit by

$$\left|a_\mu^{\rm SUSY}\right| \simeq \frac{\alpha(M_Z)}{8\pi\sin^2\theta_W}\frac{m_\mu^2}{\widetilde{m}^2}\tan\beta\left(1-\frac{4\alpha}{\pi}\ln\frac{\widetilde{m}}{m_\mu}\right), \tag{22}$$

where $\widetilde{m}$ represents a typical SUSY loop mass. (Chargino- and sneutrino-masses are assumed degenerate in that expression [52].) Also, we have included a 7–8% suppression factor due to 2-loop EW effects [34,37]. Numerically, one expects

$$\left|a_\mu^{\rm SUSY}\right| \simeq 140\times10^{-11}\left(\frac{100\ \rm GeV}{\widetilde{m}}\right)^2\tan\beta, \tag{23}$$

where $a_\mu^{\rm SUSY}$ generally has the same sign as the μ-parameter in SUSY models.

Ref. [51] found that E821 will be a stringent test of a class of supergravity models. In the minimal SU(5) SUGRA model, $\tan\beta$ is severely constrained by proton decay lifetime and no significant $a_\mu^{\rm SUSY}$ is possible. However, extended models, notably SU(5)$\times$U(1) escape that bound and can induce large effects.

Supersymmetric effects in a_μ were subsequently computed in a variety of models. Constraints on MSSM were examined in [52,53]. MSSM with large CP-violating phases was studied in [54]. Ref. [55] examined models with a superlight gravitino. Detailed studies of $a_\mu^{\rm SUSY}$ were carried out in models constrained by various assumptions on the SUSY-breaking mechanism: gauge-mediated [56,57], SUGRA [58,59,60], and anomaly-mediated [61].

If we simply employ for illustration the large $\tan\beta$ approximate formula in eq. (23) and the current constraint in eq. (20), then we find (for positive sgn(μ))

$$\tan\beta\left(\frac{100\ \rm GeV}{\widetilde{m}}\right)^2 \simeq 3.2\pm3.3. \tag{24}$$

For $\tan\beta\simeq40$, the non-trivial bound $\widetilde{m}\geq215$ GeV (95% one-sided CL) follows. It is anticipated that the uncertainty in that constraint will soon be reduced to ±1 when the E821 result is announced. One can imagine a variety of outcomes and inferences. If the central value in (24) falls to near zero, then for $\tan\beta\simeq40$, $\widetilde{m}\geq500$ GeV will result, a significant constraint. (Negative $\operatorname{sgn}\mu$ models are already tightly constrained.) (Of course, in specific models with non-degenerate gauginos and sleptons, a more detailed study is required, but here we only want to illustrate roughly the scale of supersymmetry probed.) More interesting would be the case where the central value in eq. (24) remains fixed and the error is reduced to ± 1, thereby signaling at a 3 sigma level the presence of "New Physics". A natural SUSY interpretation would be that $\operatorname{sgn}\mu$ is positive, $\tan\beta$ is large $\mathcal{O}\,(20-40)$ and $\widetilde{m} \simeq 250-350$ GeV or that $\tan\beta$ is moderate $\mathcal{O}\,(5-10)$ and $\widetilde{m} \simeq 125-180$ GeV. Either represents a very exciting prospect with important implications for collider phenomenology as well as other low energy experiments such as $b \to s\gamma$, $\mu \to e\gamma$ etc. Such scenarios are well within the mainstream of SUSY models. Hence, we anticipate a clear deviation in $a_\mu^{\rm exp}$ from Standard Model expectations to be heralded as strong evidence for supersymmetry.

3.2 Radiative Muon Mass Models

The relatively light masses of the muon and most other known fundametal fermions suggest that they may be radiatively loop induced by "New Physics" beyond the Standard Model. Although no compelling model exists, the concept is very attractive as a natural scenario for explaining the flavor mass hierarchy.

The basic idea is to start off with a naturally zero bare mass due to an underlying chiral symmetry. The symmetry is broken by quantum loop effects. They lead to a finite calculable mass which depends on the mass scales, coupling strengths and dynamics of the underlying symmetry breaking mechanism. One generically expects for the muon

$$m_\mu \propto \frac{g^2}{16\pi^2} M_F, \tag{25}$$

where g is some new interaction coupling strength and $M_F \sim 100-1000$ GeV is a heavy scale associated with chiral symmetry breaking.

Whatever source of chiral symmetry breaking is responsible for generating the muon's mass will also give rise to non-Standard Model contributions in a_μ. Indeed, fermion masses and anomalous magnetic moments are intimately connected chiral symmetry breaking operators. Remarkably, in such radiative scenarios, the additional contribution to a_μ is quite generally given by [62,63]

$$a_\mu(\text{New Physics}) \simeq C\frac{m_\mu^2}{M^2}, \qquad C \simeq \mathcal{O}\,(1)\,, \tag{26}$$

where M is some physical high mass scale associated with the "New Physics" and C is a model-dependent number roughly of order 1 (it can be larger). M need not be the same scale as M_F in eq. (25). In fact, M is usually a somewhat larger gauge or scalar boson mass responsible for mediating the chiral symmetry breaking interaction. The result in eq. (26) is remarkably simple in that it is largely independent of coupling strengths, dynamics, etc. Furthermore, rather than exhibiting the usual $g^2/16\pi^2$ loop suppression factor, a_μ(New Physics) is related to m_μ^2/M^2 by a (model dependent) constant, C, roughly of $\mathcal{O}(1)$.

To demonstrate how the relationship in eq. (26) arises, we consider a simple toy model example [63] for muon mass generation which is graphically depicted in Fig. 2.

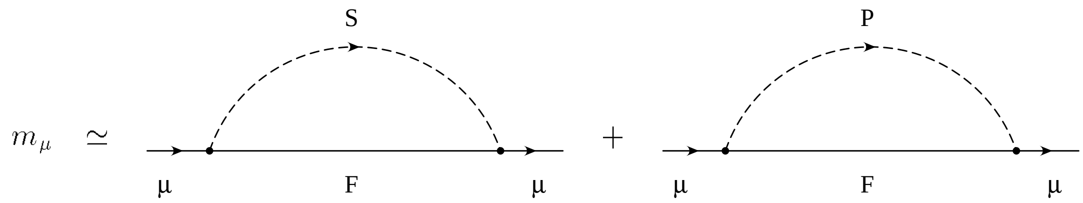

Figure 2: Example of a pair of one-loop diagrams, which can induce a finite radiative muon mass.

If the muon is massless in lowest order (i.e. no bare m_μ^0 is possible due to a symmetry), but couples to a heavy fermion F via scalar, S, and pseudoscalar, P, bosons with couplings g and $g\gamma_5$ respectively, then the diagrams give rise to

$$m_\mu \simeq \frac{g^2}{16\pi^2} M_F \ln\left(\frac{m_S^2}{m_P^2}\right). \tag{27}$$

Note that short-distance ultraviolet divergences have canceled and the induced mass vanishes in the chirally symmetric limit $m_S = m_P$.

If we attach a photon to the heavy internal fermion, F (assumed to have charge -1), then a new contribution to a_μ is also induced. For m_S, $m_P \gg M_F$, one finds [63]

$$a_\mu(\text{New Physics}) \simeq \frac{g^2}{8\pi^2}\frac{m_\mu M_F}{m_P^2}\left(\frac{m_P^2}{m_S^2}\ln\frac{m_S^2}{M_F^2} - \ln\frac{m_P^2}{M_F^2}\right). \tag{28}$$

It also vanishes in the $m_S = m_P$ chiral symmetry limit. Interestingly, a_μ(New Physics) exhibits a linear rather than quadratic dependence on m_μ at this point. Recall, that in section 1 we said that such a feature was misleading or artificial. Our subsequent discussion should clarify that point.

Although eqs. (27) and (28) both depend on unknown parameters such as g and M_F, those quantities largely cancel when we combine both expressions. One finds

$$\begin{aligned} a_\mu(\text{New Physics}) &\simeq C\frac{m_\mu^2}{m_P^2}, \\ C &= 2\left[1 - \left(1 - \frac{m_P^2}{m_S^2}\right)\ln\frac{m_S^2}{M_F^2} / \ln\frac{m_S^2}{m_P^2}\right], \end{aligned} \tag{29}$$

where C is very roughly $\mathcal{O}(1)$. It can actually span a broad range, depending on the m_S/m_P ratio. A loop produced a_μ(New Physics) effect that started out at $\mathcal{O}(g^2/16\pi^2)$ has been promoted to $\mathcal{O}(1)$ by absorbing the couplings and M_F factor into m_μ. Along the way, the linear dependence on m_μ has been replaced by a more natural quadratic dependence.

A similar relationship, a_μ(New Physics) $\simeq Cm_\mu^2/M^2$, has been found in more realistic multi-Higgs models [64], dynamical symmetry breaking scenarios such as extended technicolor [62,63], SUSY with soft masses [65], etc. It is also a natural expectation in composite models [66,67,68] or some models with large extra dimensions [69,70], although studies of such cases have not necessarily made that same connection. Basically, the requirement that m_μ remain relatively small in the presence of new chiral symmetry breaking interactions forces a_μ(New Physics) to effectively exhibit a quadratic m_μ^2 dependence.

For models of the above variety, where $|a_\mu(\text{New Physics})| \simeq m_\mu^2/M^2$, the current constraint in eq. (21) suggests (very roughly)

$$M \gtrsim \mathcal{O}(1 \text{ TeV}), \tag{30}$$

and that level of sensitivity will expand to about 4 TeV as experiment E821 improves. Of course, a non-null finding of a_μ(New Physics) $\simeq 400 \times 10^{-11}$ could be interpreted as pointing to a source of muon mass generation characterized by a mass scale of $M \sim 1-2$ TeV. Such a scale of "New Physics" could be quite natural in multi-Higgs models and soft SUSY mass scenarios. It would be somewhat low for dynamical symmetry breaking, compositeness and extra dimension models.

3.3 Other "New Physics" Examples

Many other examples of "New Physics" contributions to a_μ have been considered in the literature. General analysis in terms of effective interactions was presented in [71]. Specific examples include effects due to anomalous W boson magnetic dipole and electric quadrupole moments [72,73,74,75], muon compositeness [68], extra gauge [76] or Higgs [77] bosons, leptoquarks [78,79], bileptons [80], 2-loop pseudoscalar effects [81], compact extra dimensions [82,83] etc. If a non-Standard Model effect is uncovered, all will certainly be revisited.

4 Outlook

After many years of experimental and theoretical toil, studies of the muon anomalous magnetic moment are entering a new exciting phase. Experiment E821 at Brookhaven will soon confront theory at the $\pm 155 \times 10^{-11}$ level. Such sensitivity could start to unveil "New Physics" at the several sigma level without too much concern about theoretical hadronic uncertainties. Future analysis and runs would then confirm and refine the discovery. Theorists would have a field day. Alternatively, the experiment could confirm Standard Model expectations and tighten the bounds on "New Physics", a more traditional role for a_μ.

Stay tuned, the show is about to begin.

Acknowledgments

This work was supported by the DOE under grant number DE-AC02-98CH10886.

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