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QCD Corrections to the Reaction $p + \bar{p} \to W + \gamma + X$

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Abstract

The complete order α_s QCD corrections to the cross section and differential distributions for $p + \bar{p} \to W + \gamma + X$ are presented. At the quark level this involves the computation of the contributions from the reactions $q + \bar{q} \to W + \gamma + g$ and $q(\bar{q}) + g \to W + \gamma + q(\bar{q})$. We discuss how these corrections modify the factorization property of the lowest order amplitude and give results for future experiments at CERN and Fermilab.

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1 Introduction

The charged W-boson was discovered in $p\bar{p}$ collisions via its leptonic decay modes $W^{\pm} \to e^{\pm}\bar{\nu}_{\mu}, \mu^{\pm}\bar{\nu}_{\mu}, \tau^{\pm}\bar{\nu}_{\tau}$[1]. This particle should have both a magnetic moment μ and a quadrupole moment Q. For spin-one gauge bosons $\mu = (e/2M_W)g$, with gyromagnetic ratio $g = 2 = 1 + \kappa$. This implies that $\kappa = 1$, up to QED corrections [2] of order α. Experimental information on the magnitude of κ would be valuable in determining if the W is a gauge boson. As pointed out several years ago [3], a careful study of W-radiative processes, either in production or in decay reactions, may allow us to place a bound on κ. Typical reactions which we can use in this study are $\gamma + W \leftrightarrow q + \bar{q}, W \to q + \bar{q} + \gamma$, $\gamma + e \leftrightarrow W + \nu$ and $W \to e + \nu + \gamma$.

Presently, out of the several hundred W decay events that have been collected[4], a few radiative leptonic decays $W^{\pm} \to e^{\pm}\bar{\nu}_e\gamma$, or $W^{\pm} \to \mu^{\pm}\bar{\nu}_{\mu}\gamma$ have been reported to occur with rates compatible with QED bremsstrahlung [5]. W-decays into light mass $q\bar{q}$ pairs, such as $W^+ \to u + \bar{d}$, have been impossible to isolate from the large background of QCD high-p_T jet production processes [6]. Events which also include photons such as ($W^+ \to u + \bar{d} + \gamma$), seem impossible to resolve. However the steady increase in the number of observed W-bosons at CERN[4] and Fermilab [7], may make it possible in the near future to identify more radiative events and subsequently place a bound on the magnetic moment of the W.

The study of the angular distribution of the reaction $p + \bar{p} \to W + \gamma + X$ is probably the best way to bound the magnitude of κ. It has been known for some time that the amplitudes for $q + \bar{q} \to W + \gamma$ and $W \to l + \nu_l + \gamma$ vanish at particular kinematical values [3] reflecting the antennae patterns for radiation from classical charge distributions [8]. These zeros only occur for the gauge theory value $\kappa = 1$, so deviations from theoretical predictions are potentially interesting.

The zeros mentioned above are only exact in lowest order Born approximation in QCD, i.e., for quark-antiquark collisions in the absence of higher order gluonic corrections. Higher order QED processes also fill in the zeros but these effects should be negligible compared to those of QCD. A study of future experimental data warrants a corresponding study of higher order QCD corrections to the basic reaction. Investigations of pieces of this question have been published previously [9] but until now no complete analysis of the order α_s corrections has been reported. We present this analysis here. A corresponding calculation of the order α_s QCD corrections to the decay $W \to q + \bar{q} + \gamma$, which was much easier than this calculation, has recently been published[10].

The order α_s correction in QCD involves the analysis of several reactions. First of all the virtual and radiative gluon corrections to the reaction $q + \bar{q} \to W + \gamma$ must be calculated. The cross section for this reaction must be folded with the quark and anti-quark structure functions to calculate part of the distributions for the process $p + \bar{p} \to W + \gamma + X$. At the same order in α_s we must also calculate the reactions $q + g \to W + \gamma + q$ and $\bar{q} + g \to W + \gamma + \bar{q}$. These cross sections have to be folded with the quark/antiquark and gluon structure functions to calculate the remaining pieces of the process $p + \bar{p} \to W + \gamma + X$. The gluon radiation amplitudes have soft

(infrared) and collinear singularities which must be factored. These singular pieces are either cancelled by the virtual contribution or, via mass factorization, absorbed into the definitions of the quark, antiquark and/or gluon structure functions as well as the quark and/or antiquark fragmentation functions. Since the calculation involves many subprocesses we have split our presentation into several sections.

After a discussion of the Born amplitude in Sec.2, the QCD correction calculation is presented in Secs. 3-6. We start with the virtual corrections to the reaction $q + \bar{q} \to W + \gamma$ in Sec.3. In our calculations the quarks and gluons are massless. The hard gluon bremsstrahlung corrections are then discussed in Sec.4 and the extraction of the soft gluon bremsstrahlung pieces in Sec.5. We use dimensional regularization[11] to handle the singularities in the graphs. As a check we verify that the virtual amplitudes and the soft/collinear corrections discussed in Sec.5 satisfy the Ward identities implied by the standard model. The cross section contains double and single poles in $\epsilon = n - 4$ as well as finite terms. The singular terms are handled using the general proofs for factorization for QED and QCD[12]. Therefore the double and single pole terms must be proportional to the lowest order Born cross section.

The details of the mass factorization scheme are presented in Sec.6. At that point, all the order α_s corrections to the $q + \bar{q} \to W + \gamma$ reaction, including the contributions from $q + g \to W + \gamma + g$ and $\bar{q} + g \to W + \gamma + \bar{q}$ have been calculated. Note that portions of the singular terms must be absorbed into the structure functions and/or fragmentation functions. It is here that a degree of arbitrariness enters into the calculation and this point is discussed in some detail. Next in Sec.7 we discuss the details of the folding the parton cross sections with the quark, anti-quark and gluon structure functions.

In Sec.8 there is a discussion of results for future experiments at CERN and Fermilab. Several authors have already examined how the Born approximation photon differential cross section is modified by the addition of quark distribution functions, phenomenological p_T distribution functions for the W-boson, etc,[13][14] . Since the angular distribution of the photon in the reaction $p + \bar{p} \to W + \gamma + X$ involves the folding of angular distributions for $q + \bar{q} \to W + \gamma$ with the quark distribution functions, the original zero is transformed into a sharp dip. We are able to compare the QCD predictions for the shape of the differential cross section in this dip neglecting and including the order α_s corrections.

To give the reader an appreciation for the relative magnitudes of the various contributions we first present the photon angular distributions for the Born cross sections for $q + \bar{q} \to W + \gamma$ and $p + \bar{p} \to W + \gamma + X$. Then we add in all the QCD corrections in two different mass factorization schemes. We close this introduction with the comment that the actual prediction of the photon angular distribution for a specific experimental setup still requires further analysis. Since the W can only be unambiguosly identified from its decay products, there may be photons emitted from the charged particles present in the decay. A more detailed Monte Carlo study of the experimental difficulties, including those involved in the identification of the W and γ will be presented in a later publication.

2 The Born Amplitude

The three graphs which contribute to the Born amplitude for the reaction

$$q(p_1) + \bar{q}(p_2) \to W(q) + \gamma(k), \tag{2.1}$$

where the particle momenta are given in parenthesis, are shown in Fig.1. The charges of the quarks $q(p_1), q(p_2)$ are Q_1 and Q_2 respectively (all in units of the electric charge), while $Q = Q_1 - Q_2$ is the W-boson charge. The weak coupling constant is $g/2\sqrt{2}$ where the weak charge in the standard model is $g = 2M_W(\sqrt{2}G_F)^{1/2} = e/\sin\theta_W$. For the kinematical variables we choose $s = 2p_1 \cdot p_2\,, t = -2k \cdot p_1$ and $u = -2k \cdot p_2$. Since we work with massless quarks they are related by $s+t+u = M_W^2$. Application of the standard Feynman rules yields the following expression for the Born amplitude,

$$M^B = ie\frac{g}{2\sqrt{2}}\epsilon^\mu(q)\epsilon^\nu(k)\bar{v}(p_2)M^B_{\mu\nu}(1-\gamma_5)u(p_1), \tag{2.2}$$

where $\epsilon^\mu(q)$ and $\epsilon^\nu(k)$ denote the W-boson and photon polarization vectors respectively and

$$\begin{aligned} M^B_{\mu\nu} &= -Q_1(2p_{1\nu}\gamma_\mu - \gamma_\mu \not{k}\gamma_\nu)t^{-1} \\ &\quad +Q_2(2p_{2\nu}\gamma_\mu - \gamma_\nu \not{k}\gamma_\mu)u^{-1} \\ &\quad -2(Q_1-Q_2)[\gamma_\mu(p_1+p_2)_\nu + g_{\mu\nu}\not{k} - k_\mu\gamma_\nu](s-M_W^2)^{-1}, \end{aligned} \tag{2.3}$$

The latter expression satisfies $k^\nu M^B_{\mu\nu} = q^\mu M^B_{\mu\nu} = 0$ after using the Dirac-equation for the spinors $\bar{v}(p_2)$ and $u(p_1)$.

Since we need the cross section for (2.1) in n-dimensions [11] most of the algebra in this paper was programmed using either Form [15], Schoonschip [16] or SMP[17]. First of all we calculate the square of (2.2) summed over the initial spins and final polarizations, yielding the result

$$\begin{aligned} \sum M^B M^{B*} &= 2e^2g^2\frac{(Q_1u+Q_2t)^2}{ut(u+t)^2}(n-2) \\ &\quad \times[sM_W^2 - ut + \frac{1}{4}(n-2)(u+t)^2]. \end{aligned} \tag{2.4}$$

The factor $(Q_1u + Q_2t)$ represents the radiation zero. Notice that it can be factored from the original amplitude by rearranging the terms in (2.3) [3]and writing $M^B_{\mu\nu}$ in the so-called radiation representation [8]. Averaging over the initial quark spins and colour we find that the result for the Born cross section in n-dimensions, with $\epsilon = n-4$, is given by,

$$\begin{aligned} s^2\frac{d^2\sigma^{(0)}_{q\bar{q}}}{dtdu} &= \frac{4}{3}\frac{1+\epsilon/2}{\Gamma(1+\epsilon/2)}\pi^2 S_\epsilon \alpha g^2 (\frac{ut}{s\mu^2})^{\epsilon/2}\frac{(Q_1u+Q_2t)^2}{ut(s-M_W^2)^2} \\ &\quad \times[sM_W^2 - ut + \frac{1}{2}(1+\frac{\epsilon}{2})(u+t)^2]\delta(s+t+u-M_W^2), \end{aligned} \tag{2.5}$$

where the mass parameter μ originates from the dimensionality of the gauge coupling constants e and g in n dimensions. The constant S_ϵ, which is equal to $(4\pi)^{-2-\epsilon/2}$, has its origin in the n-dimension integration over the solid angle and α is the fine structure constant. The residues of the ϵ pole terms in the the order α_s virtual and the soft gluon corrections to (2.1) must be proportional to this expression.

3 Virtual corrections

The order α_s QCD corrections to the differential cross section (2.5) require the evaluation of the Feynman diagrams shown in Fig.2. We have calculated these graphs in n-dimensional regularization using the Feynman parametrization technique. The virtual amplitude has the following form

$$M^V = ie\frac{g}{2\sqrt{2}}\epsilon^\mu(q)\epsilon^\nu(k)\frac{\alpha_s}{\pi}C_F\bar{v}(p_2)M^V_{\mu\nu}(1-\gamma_5)u(p_1), \tag{3.1}$$

where M^V and $M^V_{\mu\nu}$ can be decomposed into two pieces

$$M^V = VM^B + R, \qquad M^V_{\mu\nu} = VM^B_{\mu\nu} + R_{\mu\nu}, \tag{3.2}$$

with

$$\begin{aligned} R_{\mu\nu} &= p_{1\nu}\gamma_\mu(Q_1A_1+Q_2A_2) + p_{2\nu}\gamma_\mu(Q_1A_3+Q_2A_4) \\ &+\gamma_\mu\not{k}\gamma_\nu Q_1A_5 + \gamma_\nu\not{k}\gamma_\mu Q_2A_6 \\ &+p_{1\mu}\gamma_\nu(Q_1A_7+Q_2A_8) + p_{2\mu}\gamma_\nu(Q_1A_9+Q_2A_{10}) \\ &+p_{1\nu}p_{2\mu}\not{k}(Q_1A_{11}+Q_2A_{12}) + p_{2\nu}p_{1\mu}\not{k}(Q_1A_{13}+Q_2A_{14}) \\ &+p_{1\mu}p_{1\nu}\not{k}(Q_1A_{15}+Q_2A_{16}) + p_{2\mu}p_{2\nu}\not{k}(Q_1A_{17}+Q_2A_{18}) \\ &+p_{1\nu}k_\mu\not{k}(Q_1A_{19}+Q_2A_{20}) + p_{2\nu}k_\mu\not{k}(Q_1A_{21}+Q_2A_{22}) \\ &+g_{\mu\nu}\not{k}_\mu(Q_1A_{23}+Q_2A_{24}) + \gamma_\nu k_\mu(Q_1A_{25}+Q_2A_{26}), \end{aligned} \tag{3.3}$$

plus terms containing an explicit factor of k_ν. The latter vanish when contracted with the lowest order amplitude due of the Ward identity $k^\nu M_{\mu\nu} = 0$ mentioned previously. In (3.1) $C_F = 4/3$ is the Casimir factor and $\alpha_s = g_s^2/4\pi$ is the QCD strong coupling constant. The function V, which multiplies the same structure as the basic Born amplitude in (2.3) contains double and single poles in ϵ which arise from the soft and collinear divergences in the virtual graphs. We find

$$\begin{aligned} V &= -e^{\epsilon/2(\gamma_E-\ln 4\pi)}\Big[\frac{2}{\epsilon^2}+\frac{1}{\epsilon}(\ln(\frac{s}{\mu^2})-\frac{3}{2}) \\ &+\frac{1}{4}\ln^2(\frac{s}{\mu^2})-\frac{3}{4}\ln(\frac{s}{\mu^2})-\frac{7}{4}\zeta(2)+2\Big], \end{aligned} \tag{3.4}$$

where γ_E is Euler's constant.Note that V does not contain ultraviolet singularities since the graphs in Fig.2 do not contribute to the renormalization of the strong, electromagnetic or weak coupling constants. We also point out that the self-energy insertions on the external quark lines vanish due to the cancellation of the collinear and ultraviolet divergences as explained in [18].

The functions $A_1 - A_{26}$, which are too lengthy to be explicitly presented here, have no poles in ϵ and contain the basic functions,

$$\begin{aligned} \mathrm{SP}(t_i,s,M_W^2) &= \mathrm{Li}_2(\frac{t_i}{M_W^2}) - \mathrm{Li}_2(\frac{M_W^2}{s}) + \ln(\frac{-t_i}{M_W^2})\ln(1-\frac{t_i}{M_W^2}) \\ &+\ln(\frac{s}{M_W^2})\Big[\ln(\frac{s}{M_W^2}-1) - \frac{1}{2}\ln(\frac{s}{M_W^2}) - \ln(\frac{-t_i}{M_W^2})\Big] + \zeta(2), \end{aligned} \tag{3.5}$$

where $i = 1,2$ and $t_1 = t, t_2 = u$. $\mathrm{Li}_2(x)$ is the standard dilogarithmic function[19] with $\zeta(2) = \mathrm{Li}_2(1) = \pi^2/6$. Other terms in $A_1 - A_{26}$ have single logarithms such as $\ln(s/M_W^2)$ and $\ln(-t_i/M_W^2)$ all multiplied by polynomials in t, u and $s - M_W^2$.

Since $M^V_{\mu\nu}$ should satisfy the Ward-identities $k^\nu M^V_{\mu\nu} = 0$ and $q^\mu M^V_{\mu\nu} = 0$, we checked that $k^\nu R_{\mu\nu} = q^\mu R_{\mu\nu} = 0$ for quarks and antiquarks whose spinors satisfy the Dirac equation. Note that the latter Ward-identity is only satisfied if we include the terms proportional to k_ν whose explicit expressions are suppressed in (3.3). Next we calculated the interference between (2.3) and (3.1) summed over the spins of the quarks and the polarization states of the photon and the W- boson. Inserting the expressions for $A_1 - A_{26}$ and using tricks to combine terms over common denominators or to regroup terms we were able to extract the expected factor $Q_1u + Q_2t$, and write the result as

$$\begin{aligned} \sum(RM^{B*} + M^BR^*) &= -\frac{1}{2}e^2g^2\frac{\alpha_s}{\pi}C_F(Q_1u+Q_2t) \\ &\times[Q_1F(t,u,s,M_W^2) + Q_2F(u,t,s,M_W^2)](s-M_W^2)^{-1}, \end{aligned} \tag{3.6}$$

where the dimensionless function

$$\begin{aligned} F(t_1,t_2,s,M_W^2) &= 4(2t_2s+2s^2+t_2^2)t_1^{-1}t_2^{-1}\mathrm{SP}(t_1,s,M_W^2) \\ &+2[4s(t_2+s)(s-M_W^2)^{-1}t_1^{-1} \\ &-2t_2s^2(s-M_W^2)^{-2}t_1^{-1} - (3t_2+2s)t_1^{-1}]\ln(\frac{s}{M_W^2}) \\ &+2[t_1s(t_2+s)^{-2} + (t_1+4s)(t_2+s)^{-1}]\ln(\frac{-t_1}{M_W^2}) \\ &+2[M_W^2(t_2^2-t_1s)(t_2+s)^{-1}(s-M_W^2)^{-1}t_1^{-1} \\ &\quad +t_2s(s-M_W^2)^{-1}t_1^{-1}]. \end{aligned} \tag{3.7}$$

Hence the order α_s virtual correction to the cross section in (2.5) is given by

$$\begin{aligned} s^2\Big(\frac{d^2\sigma^{(1)}_{q\bar{q}}}{dtdu}\Big)^V &= \frac{8}{3}\frac{1+\epsilon/2}{\Gamma(1+\epsilon/2)}\pi^2S_\epsilon\alpha g^2C_F\frac{\alpha_s}{\pi}(\frac{ut}{s\mu^2})^{\epsilon/2}\delta(s+t+u-M_W^2) \\ &\times\Big(\frac{(Q_1u+Q_2t)^2}{ut(s-M_W^2)^2}[sM_W^2 - ut + \frac{1}{2}(1+\frac{\epsilon}{2})(u+t)^2]V \\ &-\frac{1}{16}\frac{(Q_1u+Q_2t)}{(s-M_W^2)}[Q_1F(t,u,s,M_W^2) + Q_2F(u,t,s,M_W^2).]\Big) \end{aligned} \tag{3.8}$$

This result demonstrates that both the virtual corrections and the original cross section (2.5) vanish along the same line of zeros $Q_1u + Q_2t = 0$. However (3.8) does

not vanish in other regions of the t, u plane. The single and double poles which appear in expression (3.8)—via the function V— will be cancelled by corresponding terms in the soft gluon cross section as we will show in Sec.5 cf. (5.6).

4 Hard Gluon Bremsstrahlung

We now calculate the cross section for the hard gluon emission process

$$q(p_1) + \bar{q}(p_2) \to W(q) + \gamma(k) + g(k'). \tag{4.1}$$

The eight Feynman diagrams which contribute to amplitude for this process are shown in Fig.3. In the calculation of these diagrams we found it convenient to introduce eleven kinematical invariants denoted by

$$\begin{aligned}
s &= (p_1 + p_2)^2, \\
s_3 &= (k + k')^2, \\
s_4 &= (k' + q)^2, \\
s_5 &= (k + q)^2, \\
u_1 &= u = (p_2 - k)^2, \\
u_2 &= t = (p_1 - k)^2, \\
u_3 &= u' = (p_2 - k')^2, \\
u_4 &= t' = (p_1 - k')^2, \\
u_5 &= M_W^2 - s_5, \\
u_6 &= (p_1 - q)^2, \\
u_7 &= (p_2 - q)^2,
\end{aligned} \tag{4.2}$$

where $p_1 + p_2 = q + k + k'$. The invariants s, t and u are the same as those used in the calculation of the Born graphs (Sec.2) and the virtual graphs (Sec.3). Since we are considering a two $\to$ three body process only five of the invariants in (4.2) are linearly independent. The amplitude for the reaction can be written as

$$\begin{aligned}
M_a^R &= -ie\frac{g}{2\sqrt{2}} g_s T_a \epsilon^\mu(q) \epsilon^\nu(k) \epsilon^\rho(k') \\
&\quad \times \bar{v}(p_2)\Big(V^a_{\mu\nu\rho} + V^b_{\mu\nu\rho} + V^c_{\mu\nu\rho} + V^d_{\mu\nu\rho} + V^e_{\mu\nu\rho} + V^f_{\mu\nu\rho} \\
&\quad + V^g_{\mu\nu\rho} + V^h_{\mu\nu\rho}\Big)(1 - \gamma_5) u(p_1),
\end{aligned} \tag{4.3}$$

where the individual graphs a,..h, yield the third rank tensors

$$\begin{aligned}
V^a_{\mu\nu\rho} &= Q_2(2p_{2\nu} - \gamma_\nu \not{k})\gamma_\rho(\not{p}_1 - \not{q})\gamma_\mu u_1^{-1} u_6^{-1}, \\
V^b_{\mu\nu\rho} &= Q_2\gamma_\rho(\not{p}_2 - \not{k}')\gamma_\nu(\not{p}_1 - \not{q})\gamma_\mu u_3^{-1} u_6^{-1}, \\
V^c_{\mu\nu\rho} &= Q_1\gamma_\mu(\not{p}_2 - \not{q})\gamma_\rho(2p_{1\nu} - \not{k}\gamma_\nu) u_2^{-1} u_7^{-1}, \\
V^d_{\mu\nu\rho} &= Q_1\gamma_\mu(\not{p}_2 - \not{q})\gamma_\nu(\not{p}_1 - \not{k}')\gamma_\rho u_4^{-1} u_7^{-1}, \\
V^e_{\mu\nu\rho} &= Q_2(2p_{2\nu} - \gamma_\nu \not{k})\gamma_\mu(\not{p}_1 - \not{k}')\gamma_\rho u_1^{-1} u_4^{-1}, \\
V^f_{\mu\nu\rho} &= Q_1\gamma_\rho(\not{p}_2 - \not{k}')\gamma_\mu(2p_{1\nu} - \not{k}\gamma_\nu) u_2^{-1} u_3^{-1},
\end{aligned}$$

$$
\begin{aligned}
V^g_{\mu\nu\rho} &= -2Q\gamma_\rho(\not p_2 - \not k')(\gamma_\mu q_\nu + g_{\mu\nu}\not k - k_\mu\gamma_\nu)u_3^{-1}u_5^{-1}\,,\\
V^h_{\mu\nu\rho} &= 2Q(\gamma_\mu q_\nu + g_{\mu\nu}\not k - k_\mu\gamma_\nu)(\not p_1 - \not k')\gamma_\rho u_4^{-1}u_5^{-1}\,.
\end{aligned}
\tag{4.4}
$$

T_a is the $SU(3)$ colour generator whose square yields the Casimir factor $C_F = 4/3$. The square of the amplitude summed over all spins was calculated in n-dimensions using the program Form [15]. The answer was checked in 4-dimensions with Schoonschip [16]. In the extension to n-dimensions we used the fact that the axial-vector term can be treated as an external source so there is no need to extend γ_5 to n-dimensions. It is sufficient to multiply the result for the vector term by 2. The matrix elements squared for the processes $q + g \to W + \gamma + q$ and $\bar q + g \to W + \gamma + \bar q$ can be obtained from the result by crossing $p_2 \leftrightarrow -k'$ and $p_1 \leftrightarrow -k'$ respectively. Further one has to correct for an overall minus sign and to replace the colour factor C_F by T_f with the definition $\delta_{ab}T_f = \mathrm{Tr}\,T_aT_b$. In our convention $T_f = \frac{1}{2}$. Averaging over initial spins and colour the cross section (see A.14) can be written in the following form,

$$
\begin{aligned}
s^2\frac{d^2\sigma}{dtdu} &= \frac{1}{3}(\frac{1}{2})^3 S_\epsilon^2 \frac{\mu^{-\epsilon}}{\Gamma(1+\epsilon)}(\frac{ut}{s\mu^2})^{\epsilon/2}\frac{(s_4 - M_W^2)^{1+\epsilon}}{s_4^{1+\epsilon/2}}\\
&\quad\times\int_0^\pi d\theta_1 \sin^{1+\epsilon}\theta_1 \int_0^\pi d\theta_2 \sin^\epsilon\theta_2 \sum M_a^R M_a^{R*}.
\end{aligned}
\tag{4.5}
$$

In order to perform the angular integrations the square of the matrix element has to be split into terms of the type $(s')^k(s'')^l$. Here k, l are positive or negative integers and s', s'' represent pairs of the kinematical variables listed in (4.2). The decomposition has to be done in such a way that only one of the variables s', s'' contains the polar angle θ_1 and the other one has both the polar angle θ_1 and the azimuthal angle θ_2. The decomposition requires extensive partial fractioning, which exploits the following identities between the invariants defined in (4.2),

$$
\begin{aligned}
s_3 &= u_5 - u_1 - u_2\\
s_4 &= u_5 - u_6 - u_7 + M_W^2\,,\\
s_5 &= M_W^2 - u_5\,,\\
u_5 &= M_W^2 - s - u_3 - u_4\,,\\
u_6 &= M_W^2 - s - u_2 - u_4\,,\\
u_7 &= M_W^2 - s - u_1 - u_3\,.
\end{aligned}
\tag{4.6}
$$

Using these relations one can write each term in the square of the amplitude as a product of invariants where the number of invariants containing the angular integration variables is two or less. For example the term with the three denominator structure $u_3^{-1}u_4^{-1}u_5^{-1} = a_0^{-1}(u_4^{-1}u_5^{-1} + u_3^{-1}u_5^{-1} + u_3^{-1}u_4^{-1})$ where $a_0 = M_W^2 - s$. We used Schoonschip [16] to perform this decomposition. The resulting expression contains 32 distinct terms which are discussed in Appendix B. After substituting the angles

into the expressions for $(s')^k(s'')^l$ we find integrals of the form,

$$
\begin{aligned}
I_n^{(k,l)} &= \int_0^\pi d\theta_1 \sin^{n-3}\theta_1 \int_0^\pi d\theta_2 \sin^{n-4}\theta_2 (a + b\cos\theta_1)^{-k}\\
&\quad\times(A + B\cos\theta_1 + C\sin\theta_1\cos\theta_2)^{-l},
\end{aligned}
\tag{4.7}
$$

where a, b, A, B, and C are functions of the external kinematical variables s, t, u, and M_W^2. The results for the angular integrals are also listed in Appendix B. For the case of massless particles some of these integrals can be found in the literature [18][20]. However in our case they involve one particle with mass and must be calculated using a trick unless one wants to find complicated answers involving a large number of dilogarithmic functions. Our technique involved first calculating the box graph in n-dimensions and then evaluating its absorptive part. This result only involves angular integrations, so one obtains a representation for $I_n^{(k,l)}$ in terms of Feynman parameter integrals which is valid for arbitrary n and can be checked numerically. Since the denominator in Feynman parameter integrals can always be made linear in the integration variables, the resulting integrations are not too complicated. Therefore, the ϵ singularities can be extracted by integrations by parts. A typical example is the case for $k = 1, l = 1, a = -b = 1$, where

$$
\begin{aligned}
I_n^{(1,1)} &= \int_0^\pi d\theta_1 \sin^{n-3}\theta_1 \int_0^\pi d\theta_2 \sin^{n-4}\theta_2 (1 - \cos\theta_1)^{-1}\\
&\quad\times(A + B\cos\theta_1 + C\sin\theta_1\cos\theta_2)^{-1}\\
&= \frac{\epsilon\pi}{2}\frac{\Gamma(1+\epsilon)}{\Gamma(1+\epsilon/2)^2}\int_0^1 dz \frac{z^{\epsilon/2-1}(1-z)^{\epsilon/2}}{A + B - z(B + \sqrt{B^2 + C^2})}\\
&\quad\times\int_0^1 dy \frac{y^{-1+\epsilon/2}}{1 + \alpha y}
\end{aligned}
\tag{4.8}
$$

with $\alpha = (1-z)(\sqrt{B^2+C^2} - B)[A + B - z(B + \sqrt{B^2 + C^2}]^{-1}$ and $\epsilon = n - 4$. Note that in the massless limit the integrals can be checked against the answers in [18].

Expression (4.5) contains collinear singularities if $k \geq 1$ or $l \geq 1$ and $a^2 = b^2$ or $A^2 = B^2 + C^2$. These singularities arise when the gluon is emitted parallel to either the incoming quark or the incoming antiquark and they manifest themselves as pole terms in ϵ. They have to be isolated and removed via mass factorization (see Sec.6). Besides the collinear divergences mentioned above there are also infrared (I.R.) singularities arising from soft gluon emission. The photon must be detected therefore the photon energy is never soft. The soft gluon singularities show up when the mass-factorized cross section (4.5) is convoluted with the input parton distribution functions (see Sec.7). This convolution implicitly involves an integration over the variable s_4, c.f.(4.2), starting at the lower limit $s_4 = M_W^2$. The appearance of this singularity therefore depends on the power of $s_4 - M_W^2$ in (4.5). The I.R. singularity shows up when the denominator of the square of the matrix element contains the factor $(s_4 - M_W^2)^2$. A careful analysis reveals that this I.R. singularity can be traced back to the term in (4.2) which contains the product $u'^{-1}t'^{-1}$. For convenience (see

Sec.7) we split the integral over s_4 into two pieces. We define the cross section in (4.5) to be the hard gluon cross section when the integration region for s_4 starts at $s_4 = M_W^2 + \Delta$. The region of phase space where $M_W^2 < s_4 < M_W^2 + \Delta$ yields the soft gluon cross section. The parameter Δ is chosen so that $\Delta << M_W^2$ and eventually will be set to zero. It allows us to distinguish between hard and soft gluon emision. The soft gluon cross section will be calculated in the next section. The hard gluon cross section has only collinear divergences which appear as single pole terms of the form ϵ^{-1}. After the pole terms have been removed this contribution is finite and can be expressed in terms of the integrals in Appendix B in the limit that $\epsilon \to 0$.

5 Soft Gluon Corrections

The soft gluon amplitude can be obtained from the matrix element in (4.3) by applying the eikonal approximation. The limit $k' \to 0$ is taken in the numerator and denominator terms so that the kinematical invariants in the two-to-three body process can be approximated by

$$\begin{aligned} s_3 &\to 0\,, \qquad s_4 \to M_W^2\,, \quad s_5 \to s\,, \\ u_5 &\to M_W^2 - s\,, \quad u_6 \to u\,, \quad u_7 \to t\,, \end{aligned} \tag{5.1}$$

and the remaining invariants are unaltered. A close inspection of the matrix element reveals that only the $u'^{-1}t'^{-1}$ term survives in the above limit. The soft gluon matrix element can now be written in the following form,

$$M^S_{a,\rho\mu\nu} = T_a g_s \left(\frac{p_{1\rho}}{p_1 \cdot k'} - \frac{p_{2\rho}}{p_2 \cdot k'}\right) M^B_{\mu\nu}. \tag{5.2}$$

The above expression shows the factorization property which is characteristic of QED processes; in the amplitude under consideration there are no three-gluon or four-gluon couplings. The factorization property states that in the soft gluon limit each matrix element can be written as a product of a soft gluon bremsstrahlung factor and a term which represents a process without any gluons. From (2.3) and (5.2) it follows that the soft gluon amplitude equals

$$M^S_a = T_a g_s \epsilon^\rho(k') \left(\frac{p_{1\rho}}{p_1 \cdot k'} - \frac{p_{2\rho}}{p_2 \cdot k'}\right) M^B. \tag{5.3}$$

Proceeding in the same way as in the Born graph we obtain

$$\sum M^S_a M^{S*}_a = C_F g_s^2 \frac{4s}{u't'} \sum M^B M^{B*}. \tag{5.4}$$

This result is only valid for soft gluons. As before we introduce an invariant mass cutoff Δ in the phase space integrals to distinguish between soft gluons, where $s_4 < M_W^2 + \Delta$ and hard gluons, where $s_4 > M_W^2 + \Delta$. The soft gluon cross section is therefore given by

$$\begin{aligned} s^2 \left(\frac{d^2\sigma^{(1)}_{q\bar{q}}}{dtdu}\right)^S &= \frac{1}{3}\left(\frac{1}{2}\right)^3 S_\epsilon^2 \frac{\mu^{-\epsilon}}{\Gamma(1+\epsilon)} \left(\frac{ut}{s\mu^2}\right)^{\epsilon/2} \delta(s+t+u-M_W^2) \\ &\times \int_{M_W^2}^{\Delta+M_W^2} ds_4 \frac{(s_4 - M_W^2)^{1+\epsilon}}{s_4^{1+\epsilon/2}} \int_0^\pi d\theta_1 \sin^{1+\epsilon}\theta_1 \\ &\times \int_0^\pi d\theta_2 \sin^\epsilon \theta_2 \sum M^S M^{S*}. \end{aligned} \tag{5.5}$$

If $\Delta << M_W^2$ the Born matrix element $\sum M^B M^{B*}$ can be factored out of the integral. The remaining integral is over $u'^{-1}t'^{-1}$ only. This angular integral is given in the Appendix B and the subsequent integration over s_4 is straightforward. We

find

$$s^2\Big(\frac{d^2\sigma^{(1)}_{q\bar{q}}}{dtdu}\Big)^S = \frac{8}{3}\frac{1+\epsilon/2}{\Gamma(1+\epsilon/2)}\pi^2 S_\epsilon \alpha g^2 C_F \frac{\alpha_s}{\pi}\Big(\frac{ut}{s\mu^2}\Big)^{\epsilon/2} \times\delta(s+t+u-M_W^2)\frac{(Q_1u+Q_2t)^2}{ut(s-M_W^2)^2} \times\Big[sM_W^2-ut+\frac{1}{2}(1+\frac{\epsilon}{2})(u+t)^2\Big]S(s,t,u,\Delta), \tag{5.6}$$

where $S(s,t,u,\Delta)$ is the soft gluon factor defined as

$$S(s,t,u,\Delta) = e^{\epsilon/2(\gamma_E-\ln 4\pi)}\Big[\frac{2}{\epsilon^2}+\frac{1}{\epsilon}\ln\Big(\frac{\Delta^2 s}{\mu^2(s+t)(s+u)}\Big) +\frac{1}{4}\ln^2\Big(\frac{\Delta^2 s}{\mu^2(s+t)(s+u)}\Big)+\frac{1}{2}\mathrm{Li}_2\Big(\frac{ut}{(s+t)(s+u)}\Big) -\frac{3}{4}\zeta(2)\Big]. \tag{5.7}$$

Since the soft gluon cross section is proportional to the expression for the Born cross section in (2.5) it has the same properties. Hence the soft gluon corrections also vanish at the radiation zero.

Addition of the virtual contribution (3.8) and the soft contribution (5.6) to the cross section leads to the cancellation of the infrared singularities which are responsible for the double pole term ϵ^{-2}. The result is

$$\begin{aligned} s^2\Big(\frac{d^2\sigma^{(1)}_{q\bar{q}}}{dtdu}\Big)^{V+S} &= s^2\Big(\frac{d^2\sigma^{(1)}_{q\bar{q}}}{dtdu}\Big)^V + s^2\Big(\frac{d^2\sigma^{(1)}_{q\bar{q}}}{dtdu}\Big)^S \\ &= \frac{4}{3}\frac{1+\epsilon/2}{\Gamma(1+\epsilon/2)}\pi^2 S_\epsilon \alpha g^2 C_F \frac{\alpha_s}{\pi}\Big(\frac{ut}{s\mu^2}\Big)^{\epsilon/2}\delta(s+t+u-M_W^2) \\ &\times\Big(\frac{(Q_1u+Q_2t)^2}{ut(s-M_W^2)^2}\Big[sM_W^2-ut+\frac{1}{2}(1+\frac{\epsilon}{2})(u+t)^2\Big] \\ &\times[\Big(\ln\Big(\frac{\Delta^2}{(s+t)(s+u)}\Big)+\frac{3}{2}\Big)\zeta \\ &+\frac{1}{2}\ln\Big(\frac{\Delta^2}{(s+t)(s+u)}\Big)\ln\Big(\frac{s^2\Delta^2}{\mu^4(s+t)(s+u)}\Big) \\ &+\mathrm{Li}_2\Big(\frac{ut}{(s+t)(s+u)}\Big)+\frac{3}{2}\ln\frac{s}{\mu^2}+2\zeta(2)-4] \\ &-\frac{1}{8}\frac{(Q_1u+Q_2t)}{(s-M_W^2)}\Big[Q_1F(t,u,s,m_W^2)+Q_2F(u,t,s,M_W^2)\Big]\Big) \end{aligned} \tag{5.8}$$

with $\zeta = 2/\epsilon+\gamma_E-\ln 4\pi$. The collinear singularities from the initial state gluon radiation are responsible for the single pole term in (5.8). This pole term can be removed via mass factorization as will now be shown.

6 Mass-factorization

6.1 The reaction $q + \bar{q} \to W + \gamma + X$.

As has been mentioned in the introduction, the final result for the radiatively corrected cross section contains collinear divergences originating from gluon radiation from the initial quark and/or antiquark. These divergences have to be removed via mass factorization. The collinear-singular parton cross section $d\sigma_{ij}$ can be written to all orders in α_s as

$$s^2\frac{d^2\sigma_{ij}(s,t,u,\mu^2,\epsilon)}{dtdu} = \int_0^1\frac{dx_1}{x_1}\int_0^1\frac{dx_2}{x_2}\Gamma_{li}(x_1,Q^2,\mu^2,\epsilon)\Gamma_{mj}(x_2,Q^2,\mu^2,\epsilon) \times\hat{s}^2\frac{d^2\hat{\sigma}_{lm}(\hat{s},\hat{t},\hat{u},Q^2)}{d\hat{t}d\hat{u}} \tag{6.1}$$

where $\hat{s} = x_1x_2s, \hat{t} = x_1t$ and $\hat{u} = x_2u$. The Γ_{li} are the splitting functions which can be expanded as a power series in α_s, and have been calculated in the literature up to order α_s^2 [21]. They contain the collinear singularities as poles in ϵ and further depend on the mass factorization scale Q^2 which is of the order of magnitude of s,t or u. The parameter μ^2 is an artefact of n dimensional regularization which appears in the splitting function as well as the parton cross section $d\sigma_{ij}$. The $d\hat{\sigma}_{lm}$ have no collinear divergences and are therefore finite in the limit $\epsilon \to 0$. The above equation holds for the non- singlet as well as singlet case. As in the case of the Γ_{ij} all the parton cross sections $d\sigma_{ij}$ and $d\hat{\sigma}_{lm}$ can be expanded in a power series in α_s as follows:

$$s^2\frac{d^2\sigma_{ij}}{dtdu} = \sum_{n=0}^{\infty}s^2\frac{d^2\sigma_{ij}^{(n)}}{dtdu},$$
$$\hat{s}^2\frac{d^2\hat{\sigma}_{lm}}{d\hat{t}d\hat{u}} = \sum_{n=0}^{\infty}\hat{s}^2\frac{d^2\hat{\sigma}_{lm}^{(n)}}{d\hat{t}d\hat{u}}, \tag{6.2}$$

where $d\sigma_{ij}^{(n)}$ and $d\hat{\sigma}_{lm}^{(n)}$ represent the order α_s^n part of the collinear-singular and collinear-finite cross sections.

Our aim is now to calculate the coefficients in the power series expansion of the reduced cross section. Let us start with the non-singlet process $q(p_1) + \bar{q}(p_2) \to W(q) + \gamma(k) + g(k')$, whose cross section $d\sigma_{q\bar{q}}^{(1)}$ is given by the sum of (4.5) and (5.8). The splitting function has been calculated up to order α_s [22][23] and can be written as

$$\Gamma_{\bar{q}\bar{q}}(x_i,Q^2,\mu^2,\epsilon) = \Gamma_{qq}(x_i,Q^2,\mu^2,\epsilon) = \delta(1-x_i) + \frac{\alpha_s}{2\pi}[P_{qq}(x_i,\delta_i)\frac{2}{\epsilon} + f_{qq}(x_i,Q^2,\mu^2,\delta_i)], \tag{6.3}$$

where $P_{qq}(x_i,\delta_i)$ is given by

$$P_{qq}(x_i,\delta_i) = C_F[\theta(1-x_i-\delta_i)\frac{1+x_i^2}{1-x_i} + \delta(1-x_i)(2\ln\delta_i + \frac{3}{2})]. \tag{6.4}$$

The function $f_{qq}(x_i,Q^2,\mu^2,\delta_i)$ does not contain collinear divergences and can be chosen arbitrarily. This implies that the reduced cross section $d\hat{\sigma}$ depends on the way one has performed the mass factorization in (6.1). This is indicated by the mass factorization scale Q^2 which appears in $d\hat{\sigma}$ as well as in f_{qq} and the Γ_{qq}. We will come back to this point later on. In order to regulate the pole at $x_i = 1$, which appears in P_{qq} as well as in some choices for f_{qq}, we have adopted the convention in [22][24]. Here one introduces a parameter δ_i which enables us to distinguish between soft gluons, where $x_i > 1-\delta_i$, and hard gluons, where $x_i < 1-\delta_i$. The parameter δ_i is related to the quantity Δ which appears in the soft gluon factor in (5.7) via mass factorization. The exact relationship will be established in due course.

Substituting (6.3) into (6.1) we obtain the following relations between the coefficients $d\sigma_{ij}^{(n)}$ and $d\hat{\sigma}_{lm}^{(n)}$,

$$s^2\frac{d^2\hat{\sigma}_{q\bar{q}}^{(0)}(s,t,u)}{dtdu} = s^2\frac{d^2\sigma_{q\bar{q}}^{(0)}(s,t,u)}{dtdu}, \tag{6.5}$$

and

$$s^2\frac{d^2\hat{\sigma}_{q\bar{q}}^{(1)}(s,t,u)}{dtdu} = s^2\frac{d^2\sigma_{q\bar{q}}^{(1)}(s,t,u)}{dtdu} - \frac{\alpha_s}{2\pi}\Big\{\int_0^1\frac{dx_1}{x_1}[P_{qq}(x_1,\delta_1)\frac{2}{\epsilon} + f_{qq}(x_1,Q^2,\mu^2,\delta_1)] \times\hat{s}^2\frac{d^2\hat{\sigma}_{q\bar{q}}^{(0)}(x_1s,x_1t,u)}{d\hat{t}d\hat{u}} + \int_0^1\frac{dx_2}{x_2}[P_{\bar{q}\bar{q}}(x_2,\delta_2)\frac{2}{\epsilon} + f_{\bar{q}\bar{q}}(x_2,Q^2,\mu^2,\delta_2)] \times\hat{s}^2\frac{d^2\hat{\sigma}_{q\bar{q}}^{(0)}(x_2s,t,x_2u)}{d\hat{t}d\hat{u}}\Big\}. \tag{6.6}$$

Notice that we have used the relation (6.5) in deriving (6.6). From (6.3) and (6.4) we infer that P_{qq} and f_{qq} can be split into two pieces as follows,

$$P_{qq}(x_i,\delta_i) = \theta(1-x_i-\delta_i)P_{qq}^H(x_i) + \delta(1-x_i)P_{qq}^{S+V}(\delta_i) \tag{6.7}$$
$$f_{qq}(x_i,Q^2,\mu^2,\delta_i) = \theta(1-x_i-\delta_i)f_{qq}^H(Q^2,\mu^2,x_i) + \delta(1-x_i)f_{qq}^{S+V}(Q^2,\mu^2,\delta_i),$$

where P^H, f^H and P^{S+V}, f^{S+V} denote the hard and soft gluon part respectively. Substitution of (6.5), (6.6) and (6.7) into (6.2) yields

$$
\begin{aligned}
s^2\frac{d^2\hat{\sigma}^{(1)}_{q\bar{q}}(s,t,u)}{dtdu} &= s^2\frac{d^2\sigma^{(1)}_{q\bar{q}}(s,t,u)}{dtdu} \\
&\quad -\frac{\alpha_s}{2\pi}\Big\{[P^{S+V}_{qq}(\delta_1)+P^{S+V}_{\bar{q}\bar{q}}(\delta_2)]\frac{2}{\epsilon} \\
&\quad +(f^{S+V}_{qq}(Q^2,\mu^2,\delta_1)+f^{S+V}_{\bar{q}\bar{q}}(Q^2,\mu^2,\delta_2))\Big\} \\
&\quad \times s^2\frac{d^2\sigma^{(0)}_{q\bar{q}}(s,t,u)}{dtdu} \\
&\quad -\frac{\alpha_s}{2\pi}\Big\{\int_0^{1-\delta_1}\frac{dx_1}{x_1}[P^H_{qq}(x_1)\frac{2}{\epsilon}+f^H_{qq}(x_1,Q^2,\mu^2)] \\
&\quad \times\hat{s}^2\frac{d^2\sigma^{(0)}_{q\bar{q}}(x_1s,x_1t,u)}{d\hat{t}du} \\
&\quad +\int_0^{1-\delta_2}\frac{dx_2}{x_2}[P^H_{\bar{q}\bar{q}}(x_2)\frac{2}{\epsilon}+f^H_{\bar{q}\bar{q}}(x_2,Q^2,\mu^2)] \\
&\quad \times\hat{s}^2\frac{d^2\sigma^{(0)}_{q\bar{q}}(x_2s,t,x_2u)}{dtd\hat{u}}\Big\}.
\end{aligned}
\tag{6.8}
$$

From the $\delta(s+t+u-M_W^2)$ factor in the Born cross section for $d^2\sigma^{(0)}_{q\bar{q}}$ in (2.5) we infer that the first and second integrals between the curly brackets of (6.8) are evaluated for

$$
x_1=\frac{M_W^2-u}{s+t}, \qquad x_2=\frac{M_W^2-t}{s+u}, \tag{6.9}
$$

respectively, with the constraints $x_1<1-\delta_1$ and $x_2<1-\delta_2$. This implies that

$$
\delta_1<1-\frac{M_W^2-u}{s+t}, \qquad \delta_2<1-\frac{M_W^2-t}{s+u}, \tag{6.10}
$$

respectively. From the soft gluon integral in (5.5) we observe that the kinematical invariants s,t,u in the hard gluon part of $d^2\sigma^{(1)}_{q\bar{q}}$ in (6.7) satisfy the inequality, (see also Sec.4)

$$
s_4-M_W^2=s+t+u-M_W^2>\Delta. \tag{6.11}
$$

From (6.10) and (6.11) it follows that we can make the following identifications

$$
\delta_1=\frac{\Delta}{s+t}, \qquad \delta_2=\frac{\Delta}{s+u}. \tag{6.12}
$$

At this point we can cancel the ϵ^{-1} pole term appearing in $d\sigma^{(1)}_{q\bar{q}}$ and the subtraction terms in (6.8). The residue of the pole term in the latter expression is found to be

$$
R^{(1)}_{q\bar{q}} = \frac{2}{3}\pi\alpha g^2 S_\epsilon\alpha_s C_F(\frac{ut}{s\mu^2})^{\epsilon/2}
$$

$$
\begin{aligned}
&\Big\{\delta(s+t+u-M_W^2)\frac{(Q_1u+Q_2t)^2}{ut(s-M_W^2)^2}[u^2+t^2+2sM_W^2 \\
&+\epsilon(u^2+t^2+tu+sM_W^2)][3+2\ln\frac{\Delta^2}{(s+t)(s+u)}] \\
&+\Big[\frac{(Q_1(s+t)u+Q_2(M_W^2-u)t)^2}{ut(s+t)^2(su+tM_W^2)^2}\frac{(s+t)^2+(M_W^2-u)^2}{(s+t+u-M_W^2)(M_W^2-u)^2} \\
&\{u^2(s+t)^2+t^2(M_W^2-u)^2+2(M_W^2-u)(s+t)sM_W^2 \\
&+\epsilon[u^2(s+t)^2+t^2(M_W^2-u)^2+(ut+sM_W^2)(M_W^2-u)(s+t)]\}\Big] \\
&+[u\leftrightarrow t, Q_1\leftrightarrow Q_2]\Big\}.
\end{aligned}
\tag{6.13}
$$

However as has been already pointed out in the text after (6.4), $d\hat{\sigma}^{(1)}_{q\bar{q}}$ depends on the choice we make for the function f_{qq}. In the literature the following choices have been made (for a review see [25]).

A. Minimal Subtraction (MS).

Here one subtracts the pole terms whose residue is given by (6.13). This also includes

$$
f_{qq}(x_i,Q^2,\mu^2,\delta_i)=\ln\frac{Q^2}{\mu^2}\,P_{qq}(x_i,\delta_i). \tag{6.14}
$$

B. Modified Minimal Subtraction ($\overline{\rm MS}$). In this scheme one subtracts both the pole term as well as the constant $\gamma_E-\ln 4\pi$, which is a typical artifact of n-dimensional regularization, by choosing

$$
f_{qq}(x_i,Q_i^2,\mu^2,\delta_i)=(\gamma_E-\ln 4\pi+\ln\frac{Q^2}{\mu^2})P_{qq}(x_i,\delta_i). \tag{6.15}
$$

This choice implies that $\gamma_E-\ln 4\pi$ is completely removed from $d\hat{\sigma}^{(1)}_{q\bar{q}}$.

C. DIS subtraction scheme.

One subtracts the non-pole term found in the parton structure function in deep inelastic lepton quark scattering [22][23], by choosing

$$
\begin{aligned}
f_{qq}(x_i,Q^2,\mu^2,\delta_i) &= P_{qq}(x_i,\delta_i)(\gamma_E-\ln 4\pi+\ln\frac{Q^2}{\mu^2}) \\
&\quad +C_F\Big\{\Big(\frac{1+x_i^2}{1-x_i}\ln\frac{1-x_i}{x_i}+3+2x_i-\frac{3}{2}\frac{1}{1-x_i}\Big)\theta(1-x_i-\delta_i) \\
&\quad +\delta(1-x_i)[\ln^2\delta_i-\frac{3}{2}\ln\delta_i-\frac{9}{2}-2\zeta(2)]\Big\}.
\end{aligned}
\tag{6.16}
$$

This is a natural choice because the phenomenological quark and/or anti-quark distribution functions used in many calculations are determined by deep-inelastic lepton-hadron scattering data.

D. DY subtraction scheme.

One subtracts the non-pole term found in the Drell-Yan cross section for quark-antiquark annihilation into the vector bosons of the standard model, in particular into the W-boson [22][23].

$$\begin{aligned} f_{qq}(x_i,Q^2,\mu^2,\delta_i) &= P_{qq}(x_i,\delta_i)(\gamma_E - \ln 4\pi + \ln\frac{Q^2}{\mu^2}) \\ &\quad +C_F\Big\{\Big(\frac{1+x_i^2}{1-x_i}\ln\frac{(1-x_i^2)}{x_i}\Big)\theta(1-x_i-\delta_i) \\ +\delta(1-x_i) &\qquad [2\ln^2\delta_i - 4 + 2\zeta(2)]\Big\} \end{aligned} \tag{6.17}$$

Notice that the $p+\bar{p}\to W+\gamma+X$ reaction under consideration belongs to the same class as Dreil-Yan $p+\bar{p}\to W+X$ reaction except that there is one additional photon in the final state. Therefore this would be a good choice if the quark parton distribution functions are extracted from lepton pair production rather than from lepton-hadron scattering.

One should realize that the subtraction schemes mentioned above only make sense if the renormalization-group improved quark-distribution functions include the second order contributions to the anomalous dimension and the latter have been determined in the same subtraction scheme as the reduced cross section. In order to minimize our sensitivity to the choice of the various subtraction schemes one has several options. One can manipulate the mass factorization scale Q^2 according to the principle of minimum sensitivity (P.M.S.)[26] or fastest asymptotic convergence (F.A.C.) [27]. One can also calculate the ratio of two quantities which have been computed in the same subtraction scheme. The last procedure will work even better if the quantities belong to the same type of process. In this way the dependence on the mass factorization of the ratio will be less than in the separate quantities. We will choose the last option since it is the most convenient. For that purpose we compare the process $p+\bar{p}\to W+\gamma+X$ with $p+\bar{p}\to W+X$. Both reactions are described by the Drell-Yan mechanism. In the next section we will therefore compute the quantity

$$K = \frac{d\sigma(p+\bar{p}\to W+\gamma+X)}{\sigma^W_{\rm tot}(p+\bar{p}\to W+X)}, \tag{6.18}$$

where the explicit expressions for $d\sigma(p+\bar{p}\to W+\gamma+X)$ and $\sigma^W_{\rm tot}(p+\bar{p}\to W+X)$ will be presented in (7.8) and (7.15) respectively. We will first choose the mass factorization as given in D with the mass factorisation scale Q^2 equal to s. This subtraction scheme implies that $\sigma^W_{\rm tot}(p+\bar{p}\to W+X)$ does not have any QCD

corrections in order α_s so that this process is very easy to compute. The subtraction term for $d\sigma^{(1)}_{qq}$ can be obtained by substitution of (6.17) into (6.8). The soft-plus-virtual gluon portion of the reduced cross section $d\hat{\sigma}^{(1)}_{qq}$ has the simple form,

$$\begin{aligned} s^2\Big(\frac{d\hat{\sigma}^{(1)}_{q\bar{q}}}{dtdu}\Big)^{S+V} &= \frac{1}{24}\alpha g^2 C_F\frac{\alpha_s}{\pi}\delta(s+t+u-M_W^2) \\ &\quad \Big\{\frac{(Q_1u+Q_2t)^2}{ut(s-M_W^2)^2}(t^2+u^2+2sM_W^2) \\ &\quad \times\Big[-\frac{1}{2}\ln^2\frac{s+u}{s+t}+{\rm Li}_2\Big(\frac{ut}{(s+t)(s+u)}\Big)\Big] \\ &\quad -\frac{1}{4}\frac{(Q_1u+Q_2t)}{(s-M_W^2)}(Q_1F(t,u,s,M_W^2)+Q_2F(u,t,s,M_W^2))\Big\}. \end{aligned} \tag{6.19}$$

The hard gluon part of the reduced cross section $d\hat{\sigma}^{(1)}_{qq}$ is rather long and cannot be given in full. However for convenience in discussing the different factorization schemes we give an explicit answer for the finite term that results from the cancellation of the pole terms. The rest of the answer can be expressed in terms of integrals which are evaluated using the results in Appendix B from which all the pole terms in ϵ are dropped. The final result is therefore

$$\begin{aligned} &s^2\Big(\frac{d\hat{\sigma}^{(1)}_{q\bar{q}}}{dtdu}\Big)^H = \frac{1}{48}\alpha g^2\frac{\alpha_s}{\pi}C_F\Big\{[Q_1(s+t)u+Q_2(M_W^2-u)t]^2 \\ &\times[u^2(s+t)^2+t^2(M_W^2-u)^2+2sM_W^2(M_W^2-u)(s+t)] \\ &\Big[[(s+t)^2+(M_W^2-u)^2]\ln\frac{(s+t)(M_W^2-u)}{s(s+t+u)}+(s+t+u-M_W^2)^2\Big] \\ &\times u^{-1}t^{-1}(s+t)^{-2}(M_W^2-u)^{-2}(tM_W^2+su)^{-2}(s+t+u-M_W^2)^{-1} \\ &+[u\leftrightarrow t, Q_1\leftrightarrow Q_2]\Big\} \\ &+\frac{1}{3}\Big(\frac{1}{2}\Big)^{10}\frac{1}{\pi}\alpha g^2\frac{\alpha_s}{\pi}C_F\frac{s+t+u-M_W^2}{s+t+u}\Big(\int d\Omega_n|M^{q\bar{q}g}|^2\Big)^{\rm finite}, \end{aligned} \tag{6.20}$$

where the $(\,)^{\rm finite}$ denotes the finite part of the angular integrals listed in the Appendix B. If one wants to use another mass factorization (i.e. A, B, or C) then one can add the difference between them and D via the mass factorization formulae in (6.3) to (6.19) and (6.20). We will present results for scheme C later in the paper.

6.2 The reaction $q + g \to W + \gamma + q$ and $\bar{q} + g \to W + \gamma + \bar{q}$.

The calculation of the correction term for the differential cross sections for the reactions mentioned in Sec.4, namely $q + g \to W + \gamma + q$ and $\bar{q} + g \to W + \gamma + \bar{q}$, require modifications due to the presence of the gluon in the initial state, and a quark/antiquark in the final state. The first difference is that the splitting function Γ_{li} in (6.1) now belongs to the singlet class. The second difference is due to the fragmentation of the outgoing (anti) quark into a (anti) quark and a photon. The latter process leads to collinear divergences in the final state which have to be removed via the introduction of fragmentation functions defined by $D_{ij}(x, Q^2, \mu^2, \epsilon)$. Consequently the mass factorization as stated in (6.1) has to be modified in the following way [28]

$$\begin{aligned} s^2\frac{d^2\sigma_{ij,\gamma}(s,t,u,\mu^2,\epsilon)}{dtdu} &= \int_0^1 \frac{dx_1}{x_1}\int_0^1 \frac{dx_2}{x_2}\Gamma_{li}(x_1,Q^2,\mu^2,\epsilon)\Gamma_{mj}(x_2,Q^2,\mu^2,\epsilon) \\ &\quad \times \hat{s}^2\frac{d^2\hat{\sigma}_{lm,\gamma}(\hat{s},\hat{t},\hat{u},Q^2)}{d\hat{t}d\hat{u}} \\ &\quad + \int_0^1 \frac{dx_1}{x_1}\int_0^1 \frac{dx_2}{x_2}\int_0^1 \frac{dx_3}{x_3^2}\Gamma_{li}(x_1,Q^2,\mu^2,\epsilon)\Gamma_{mj}(x_2,Q^2,\mu^2,\epsilon) \\ &\quad \times \hat{s}^2\frac{d^2\hat{\sigma}^{DY}_{lm,n}(\hat{s},\hat{t},\hat{u},Q^2)}{d\hat{t}d\hat{u}}D_{\gamma n}(x_3,Q^2,\mu^2,\epsilon)\,. \end{aligned} \tag{6.21}$$

From the above we infer that the mass factorization not only becomes multiplicative but also additive. This implies that besides the reduced cross section $d^2\hat{\sigma}_{lm}$ which stands for the process in (2.1) we have to introduce another one defined by $d^2\hat{\sigma}^{DY}_{lm,n}$. The latter process, which is also free from collinear divergences, represents the differential Drell-Yan (DY) cross section with the partons l, m in the initial state and a quark n in the final state which fragments into a photon accompanied by another quark.

This fragmentation process is not due to strong interactions but to QED. Since we deal with massless quarks naive perturbation theory is not applicable. Collinear divergences occur in electromagnetic processes when the photon is emitted parallel to the quark. These divergences are removed via mass factorization. In this way the reduced cross section $d\hat{\sigma}_{lm}$ also depends on the subtraction scheme of the final state collinear divergences. This introduces an extra arbitrariness on top of the subtraction scheme dependences due to the initial state collinear divergences and the strong coupling constant renormalization. As in the QCD case, one can fix the subtraction scheme for photon final state processes by comparing them with similar reactions where the same fragmentation function D_{ij} appears. Examples of these reactions are $q + q \to q + q + \gamma$ (direct photon production) and $e^+ + e^- \to q + \bar{q} + \gamma$ (see Ref.29).

From the experimental viewpoint it is very hard to extract information about the photonic part of the above processes. This is mainly due to the small electromagnetic coupling constant and the presence of other competing processes (such

as π^0 decay) which give rise to photons in the final state. Various mass factorization schemes can yield small effects in direct photon production but lead to large corrections to the second part of (6.21) and, therefore, significantly modify $d^2\hat{\sigma}_{lm}$. For the intial state gluon processes mentioned above we encounter the (anti) quark gluon splitting function. Up to order α_s this is given by

$$\Gamma_{qg}(x_i,Q^2,\mu^2,\epsilon) = \Gamma_{\bar{q}g}(x_i,Q^2,\mu^2,\epsilon) = \frac{\alpha_s}{2\pi}[P_{qg}(x_i)\frac{2}{\epsilon} + f_{qg}(x_i,Q^2,\mu^2)]. \tag{6.22}$$

Notice that the integrated quantities are free from IR divergences because there are no gluons in the final state and therefore the introduction of a parameter like δ_i in (6.3) is not needed. The function P_{qg} is given by

$$P_{qg}(x_i) = T_f[x_i^2 + (1-x_i)^2]\,, \tag{6.23}$$

and the non-pole term $f_{qg}(x_i)$ is determined by the subtraction scheme. The photon fragmentation function $D_{\gamma q}$ is defined in an analogous way.

$$D_{\gamma q}(x_i,Q^2,\mu^2,\epsilon) = D_{\gamma\bar{q}}(x_i,Q^2,\mu^2,\epsilon) = \frac{\alpha}{2\pi}Q_i^2[P_{\gamma q}(x_i)\frac{2}{\epsilon} + f_{\gamma q}(x_i,Q^2,\mu^2)], \tag{6.24}$$

where $P_{\gamma q}$ is equal to the abelian part of P_{gq} [21][29]

$$P_{\gamma q}(x_i) = \frac{1+(1-x_i)^2}{x_i}, \tag{6.25}$$

and $f_{\gamma q}$ depends on the chosen subtraction scheme.

In order to determine the order α_s contribution to $d^2\hat{\sigma}_{lm,\gamma}$ we also need the order α_s quark gluon initial state DY cross section. The Feynman diagrams for the reaction $q(p_1) + g(k') \to W(q) + q(p_2)$ are shown in Fig.4. The corresponding cross section is given by

$$\begin{aligned} s^2\frac{d^2\sigma^{DY}_{qg,q}}{dtdu} &= -\frac{4}{3}\frac{(1+\epsilon/2)}{\Gamma(1+\epsilon/2)}\pi^2 S_\epsilon g^2 T_f \alpha_s(\frac{ut}{s\mu^2})^{\epsilon/2}\frac{1}{su} \\ &\quad [tM_W^2 - us + \frac{1}{2}(1+\frac{\epsilon}{2})(s+u)^2]\delta(s+t+u-M_W^2), \end{aligned} \tag{6.26}$$

where $s = (p_1+k')^2, t = (p_1-p_2)^2$ and $u = (k'-p_2)^2$. This result can be derived from the expression for the matrix element squared in (2.4) by substituting $Q_1 = Q_2$, replacing e by g_s and interchanging s and t. Further we have to introduce a colour factor T_f and change the overall sign. The DY cross section for $\bar{q} + g \to \bar{q} + W$ follows from the result (6.26) by interchanging u and t.

As discussed above the expressions for the non-pole functions f_{qg} and $f_{\gamma q}$ depend on the chosen mass factorization. For f_{qg} we can proceed in a manner analogous to the case of $f_{q\bar{q}}$ in (6.14)-(6.17). The MS and the $\overline{\rm MS}$ schemes are given by the same expressions as in (6.14) and (6.15) where P_{qq} is replaced by P_{qg}. For case C,

cf.(6.16), we take the f_{qg} which is obtained in deep inelastic lepton-gluon scattering [22][23],

$$\begin{aligned} f_{qg}(x_i,Q^2,\mu^2) &= f_{\bar{q}g}(x_i,Q^2,\mu^2) \\ &= P_{qg}(x_i)(\gamma_E - \ln 4\pi + \ln\frac{Q^2}{\mu^2}) \\ &\quad +T_f\Big\{[x_i^2+(1-x_i)^2]\ln(\frac{1-x_i}{x_i}) + 6x_i(1-x_i)\Big\}. \end{aligned} \tag{6.27}$$

Finally we can also determine f_{qg} from the integrated DY cross section in (6.26) (see case D in (6.17)). Here we find [22][23]

$$\begin{aligned} f_{qg}(x_i,Q^2,\mu^2) &= f_{\bar{q}g}(x_i,Q^2,\mu^2) \\ &= P_{qg}(x_i)(\gamma_E - \ln 4\pi + \ln\frac{Q^2}{\mu^2}) \\ &\quad +T_f\Big\{[x_i^2+(1-x_i)^2]\ln\frac{(1-x_i)^2}{x_i} + 1 + \frac{1}{2}(1-x_i)(1+3x_i)\Big\}. \end{aligned} \tag{6.28}$$

As in the quark-antiquark initial state reaction we can decrease the sensitivity of the QCD radiative corrections on the chosen subtraction scheme by computing the quantity K in (6.18). In the next section we compute K in the subtraction scheme given by (6.28),(case D). In this case the correction to $\sigma_{\rm tot}^W$ coming from the $qg, \bar{q}g$ subprocesses will be zero. We also examine the differences between schemes C and D.

For the fragmentation function $f_{\gamma q}(= f_{\gamma\bar{q}})$ we can proceed in a way analogous to our treatment of processes with collinear singularities in the initial state. However as has been explained before the choice of a convenient subtraction scheme is more difficult than in the two previous cases. The main problem is in finding a suitable process where one can fix the subtraction of the collinear divergences in the photon fragmentation function $D_{\gamma q}$. From the phenomenological viewpoint it is harder to determine the QED radiative corrections to a strong process than the QCD radiative corrections to an electro-weak reaction. In this paper we have simply chosen the $\overline{\rm MS}$ scheme for $D_{\gamma q}$. An alternative approach would be to follow the approach in [29] and choose the fragmentation function determined in the process $e^+ + e^- \to q + \bar{q} + \gamma$, namely

$$\begin{aligned} f_{\gamma q}(x_i,Q^2,\mu^2) &= f_{\gamma\bar{q}}(x_i,Q^2,\mu^2) \\ &= P_{\gamma q}(x_i)(\gamma_E - \ln 4\pi + \ln\frac{Q^2}{\mu^2}) \\ &\quad +\frac{1+(1-x_i)^2}{x_i}(\ln(1-x_i)+\frac{3}{2}). \end{aligned} \tag{6.29}$$

This subtraction scheme has also been used in the calculation of direct photon production processes such as $q + q \to q + q + \gamma$. However in this paper we have chosen the $\overline{\rm MS}$ expression for $f_{\gamma q}$, ie., scheme B.

Notice that in contrast to the QCD radiative corrections we cannot use the same quantity K as defined in (6.18) in order to diminish the sensitivity of the reduced cross section $d\hat{\sigma}_{\gamma q}$ to the QED mass factorization. From (6.22)-(6.24) we infer the $O(\alpha_s)$ correction to the lowest order process in (2.1) due to initial state (anti) quark gluon reactions is

$$\begin{aligned} s^2\frac{d^2\hat{\sigma}_{gq}^{(1)}(s,t,u)}{dtdu} &= s^2\frac{d^2\sigma_{gq}^{(1)}(s,t,u)}{dtdu} - \frac{\alpha_s}{2\pi}\int_0^1\frac{dx_2}{x_2}\Big[P_{\bar{q}g}(x_2)\frac{2}{\epsilon} + f_{\bar{q}g}(x_2,Q^2,\mu^2)\Big] \\ &\quad \times\hat{s}^2\frac{d^2\sigma_{gq,\gamma}^{(0)}(x_2s,t,x_2u)}{dtd\hat{u}} \\ &\quad -\frac{\alpha}{2\pi}Q_q^2\int_0^1\frac{dx_3}{x_3^2}\Big[P_{\gamma q}(x_3)\frac{2}{\epsilon} + f_{\gamma q}(x_3,Q^2,\mu^2)\Big] \\ &\quad \times\hat{s}^2\frac{d^2\hat{\sigma}_{gq,q}^{(1)DY}(s,t/x_3,u/x_3)}{d\hat{t}d\hat{u}} \end{aligned} \tag{6.30}$$

For the convolution of the photon fragmentation function with the DY cross section we need the following relations

$$\hat{s} = s, \quad \hat{t} = \frac{t}{x_3}, \quad \hat{u} = \frac{u}{x_3}. \tag{6.31}$$

Here x_3 is the fraction of the momentum of the outgoing quark which is carried away by the photon. The corresponding relations for the convolution of the splitting function with the lowest order cross section $d^2\sigma_{qq}^{(0)}$ were given in the text after (6.1) i.e., $\hat{s} = x_1x_2s, \hat{t} = x_1t$ and $\hat{u} = x_2u$.

An important remark we want to make concerns the factor $(\hat{u}\hat{t}/\hat{s}\mu^2)^{\epsilon/2}$ which appears in front of the lowest order α and α_s cross sections $d\sigma_{qq}^{(0)}$ and $d\sigma_{gq,q}^{(1)DY}$. It does not appear in the convolution with the splitting and fragmentation functions. This is clear for the convolution with the splitting function because the relations in (6.2) ensure that $(\hat{u}\hat{t}/\hat{s}\mu^2)^{\epsilon/2} = (ut/s\mu^2)^{\epsilon/2}$. However this is not obvious for the fragmentation function where $(\hat{u}\hat{t}/\hat{s}\mu^2)^{\epsilon/2} = (ut/s\mu^2x_3^2)^{\epsilon/2}$. Using another regularization method for the final state collinear divergences where the dimension of space-time n is taken to be four (e.g., where the photon has a mass λ), we observe that the replacement $(\hat{u}\hat{t}/\hat{s}\mu^2)^{\epsilon/2} \to (ut/s\mu^2x_3^2)^{\epsilon/2}$ is wrong. Instead we have the same transformation as in the case of the initial state collinear divergences i.e. $(\hat{u}\hat{t}/\hat{s}\mu^2)^{\epsilon/2} \to (ut/s\mu^2)^{\epsilon/2}$. Therefore, we have to consider this factor as a redundant variable which is an artifact of n-dimensional regularization or reduction. It will eventually drop out in the expression for the reduced cross section.

The residue of the pole term of the quark gluon initial state process is given by

$$\begin{aligned} R_{qg}^{(1)} &= \frac{2}{3}\pi\alpha g^2 S_\epsilon\alpha_s T_f(\frac{ut}{s\mu^2})^{\epsilon/2}\Big\{[Q_1(M_W^2-t)u + Q_2(s+u)t]^2 \\ &\quad \times[(M_W^2-t)^2u^2 + (s+u)^2t^2 + 2M_W^2s(s+u)(M_W^2-t) \\ &\quad +\epsilon[(M_W^2-t)^2u^2 + (s+u)^2t^2 + (ut+sM_W^2)(M_W^2-t)(s+u)]\Big] \\ &\quad \times[(s+u)^2 - 2(M_W^2-t)(s+u) + 2(M_W^2-t)^2] \end{aligned}$$

$$
\begin{aligned}
&\times(s+u)^{-3}(M_W^2-t)^{-2}(M_W^2u+st)^{-2}u^{-1}t^{-1}\\
&+Q_2^2\Big[s^2(t+u)^2+u^2(M_W^2-s)^2+2M_W^2t(u+t)(M_W^2-s)\\
&+\epsilon[s^2(t+u)^2+u^2(M_W^2-s)^2+(su+tM_W^2)(u+t)(M_W^2-s)]\Big]\\
&\times[2(M_W^2-s)^2-2(M_W^2-s)(u+t)+(u+t)^2]\\
&(u+t)^{-3}(M_W^2-s)^{-2}u^{-1}s^{-1}\Big\}. \qquad (6.32)
\end{aligned}
$$

The reduced cross section $d\hat{\sigma}_{gq}^{(1)}$ for the gluon quark initial state reaction now follows from (6.30) and (6.32). We find the analogous result to (6.20),

$$
\begin{aligned}
&s^2\Big(\frac{d\hat{\sigma}_{qg}^{(1)}}{dtdu}\Big)^H=\frac{1}{48}\alpha g^2\frac{\alpha_s}{\pi}T_f\Big\{[Q_1(M_W^2-t)u+Q_2(s+u)t]^2\\
&\times[u^2(M_W^2-t)^2+t^2(s+u)^2+2(M_W^2-t)(s+u)sM_W^2]\\
&\times[(s+u)^2-2(M_W^2-t)(s+u)+2(M_W^2-t)^2]\ln\frac{(s+u)(M_W^2-t)}{s(s+t+u)}\\
&-\frac{1}{2}(s+u)^2-(M_W^2-t)(s+u)+\frac{3}{2}(M_W^2-t)^2]\\
&\times u^{-1}t^{-1}(s+u)^{-3}(M_W^2-t)^{-2}(uM_W^2+st)^{-2}\\
&+Q_2^2[s^2(u+t)^2+u^2(M_W^2-s)^2+2tM_W^2(M_W^2-s)(u+t)]\\
&\times\Big[[2(M_W^2-s)^2-2(M_W^2-s)(u+t)+(u+t)^2]\ln\frac{(s+t+u-M_W^2)^2}{s(s+t+u)}+(u+t)^2\Big]\\
&\times s^{-1}u^{-1}(u+t)^{-3}(M_W^2-s)^{-2}\Big\}\\
&+\frac{1}{3}\Big(\frac{1}{2}\Big)^{10}\frac{1}{\pi}\alpha g^2\frac{\alpha_s}{\pi}T_f\frac{s+t+u-M_W^2}{s+t+u}\Big(\int d\Omega_n|M^{qg}|^2\Big)^{\text{finite}}.
\end{aligned}
\qquad (6.33)
$$

This form explicitly presents the remainder from the cancellation of the pole terms so that other factorization schemes can be investigated if necessary. The second term containing the integral is to be evaluated from the results in Appendix B after dropping all the pole terms in ϵ. The reduced cross section $d\hat{\sigma}_{g\bar{q}}^{(1)}$ for the gluon anti-quark initial state reaction can be obtained from expression (6.33) by interchanging $Q_1 \leftrightarrow Q_2$ and $u \leftrightarrow t$. Finally we conclude this section with the remark that the reduced cross sections derived here were also computed using n-dimensional reduction. We found the same results as those obtained with the n-dimensional regularization method.

7 The $p\bar{p}$ reaction

In this section we will discuss the hadron induced two-particle-exclusive reaction

$$
p(P_1)+\bar{p}(P_2)\to W(q)+\gamma(k)+X, \qquad (7.1)
$$

where p and $\bar{p}$ denote the proton and antiproton respectively. The quantity X stands for all final hadronic states which we sum over so that the above process is inclusive with respect to the outgoing hadrons. We use capital letters for the momenta and invariants of the proton and antiproton to distinguish them from those of the quarks and gluons.

We are primarily interested in how the radiation zeros in the reaction $q+\bar{q}\to W+\gamma$ are altered by the QCD corrections which we have calculated in the previous sections. However, we note that the radiation zero does not appear in the lowest order contribution to (7.1) due to the smearing by the quark structure functions[3][13][14]. Both contributions can now be studied separately.

The hadronic process in (7.1) is described by the following cross section,

$$
S^2\frac{d^2\sigma(S,T,U)}{dTdU}=\int_{x_{1-}}^1\frac{dx_1}{x_1}\int_{x_{2-}}^1\frac{dx_2}{x_2}H_{lm}(x_1,x_2,Q^2)s^2\frac{d^2\hat{\sigma}_{lm}(s,t,u,Q^2)}{dtdu}, \qquad (7.2)
$$

where the lower limits are

$$
x_{1-}=(M_W^2-U)/(S+T), \qquad x_{2-}=(M_W^2-x_1T)/(x_1S+U),
$$

respectively. Here $d^2\hat{\sigma}_{lm}$ denotes the reduced (mass factorized) parton cross section in (6.2) which has been derived up to order α_s in the previous section. S, T and U denote the square of the hadron C.M. energy and the square of the momentum transfer of the photon with respect to the proton and anti-proton respectively. They are defined by

$$
S=(P_1+P_2)^2, \quad T=(P_1-k)^2, \quad U=(P_2-k)^2. \qquad (7.3)
$$

The quantities x_1, x_2 stand for the fractions of the momenta of the incoming hadrons which are carried away by partons l and m respectively. The parton kinematical variables now become

$$
s=x_1x_2S, \qquad t=x_1T, \qquad u=x_2U. \qquad (7.4)
$$

The integration boundaries in (7.2) are derived from the conditions (see (A.8) and (A.11)),

$$
\begin{aligned}
&0<x_1<1, \qquad 0<x_2<1,\\
&s_4=x_1x_2S+x_1T+x_2U\geq M_W^2, \qquad (7.5)
\end{aligned}
$$

(see also the Dalitz plot in Fig.5). Finally $H_{lm}(x_1,x_2,Q^2)$ is a product of the scale dependent parton distribution functions $f_l^H(x,Q^2)$. It is given by

$$
H_{lm}(x_1,x_2,Q^2)=f_l^p(x_1,Q^2)f_m^{\bar{p}}(x_2,Q^2) \qquad (7.6)
$$

where Q^2 is the mass factorization scale defined below (6.1). As mentioned in the text after (6.18) we choose $Q^2 = s$, where s is given in (7.4). Notice that the normalization scale also appears in the running coupling constant $\alpha_s(Q^2)$, where we also set $Q^2 = s$. The boundaries of the integrals in (7.2) have to be modified when one considers processes where there are gluons in the final state. This situation will be described later. Let us first start with the Born contribution. In this case $d^2\hat{\sigma}$ is proportional to a δ-function and we can write

$$s^2 \frac{d^2\hat{\sigma}^{(0)}_{q\bar{q}}}{dtdu} = \delta(s+t+u-M_W^2)\sigma^{(0)}_{q\bar{q}}(s,t,u), \tag{7.7}$$

where $\sigma^{(0)}_{q\bar{q}}$ can be obtained from (2.5) by taking the limit $\epsilon \to 0$. The Born contribution to the hadronic cross section becomes

$$\begin{aligned} S^2\Big(\frac{d^2\sigma(S,T,U)}{dTdU}\Big)^B &= \int_{x_{1-}}^1 \frac{dx_1}{x_1}\frac{1}{M_W^2 - x_1T} \\ &\times H_{q\bar{q}}\Big(x_1, \frac{M_W^2 - x_1T}{x_1S+U}, \frac{x_1(M_W^2-x_1T)}{x_1S+U}S\Big) \\ &\times \sigma^{(0)}_{q\bar{q}}\Big(\frac{x_1(M_W^2-x_1T)}{x_1S+U}S, x_1T, \frac{(M_W^2-x_1T)}{x_1S+U}U\Big). \end{aligned} \tag{7.8}$$

In the case where there are gluons in the final state we have to split the integration region over x_2 in (7.2) into two parts. Following the discussion at the end of Sec.4 and the beginning of Sec.6, we distinguish the soft gluon region

$$\frac{M_W^2 - x_1T}{x_1S+U} < x_2 < \frac{\Delta + M_W^2 - x_1T}{x_1S+U}, \qquad M_W^2 < s_4 < M_W^2 + \Delta, \tag{7.9}$$

and the hard gluon region, which is given by

$$\frac{\Delta + M_W^2 - x_1T}{x_1S+U} < x_2 < 1, \qquad s_4 > M_W^2 + \Delta. \tag{7.10}$$

In the soft gluon region one can approximate the mass singular cross section $d\sigma_{q\bar{q}}$ given in (4.5) by a δ-function, multiplied by the same factor as appears in (5.6). This δ-function follows from the overall four momentum $\delta^{(n)}$-function in the phase space factor of (A.10) where we have neglected the gluon momentum k' with respect to the other momenta involved. In this way one obtains the soft gluon contribution to the parton cross section which is given by (5.6). Due to the upper boundary of (7.9) and the singular behaviour of (5.6) when $s_4 \to M_W^2$ this contribution depends logarithmically on Δ.

After addition of the virtual gluon contribution to the cross section i.e., (3.8), which is also proportional to the same δ-function, one obtains an infrared finite quantity (c.f.(5.8)). Mass factorization then leads to the collinear finite quantity in (6.19),

$$s^2\Big(\frac{d^2\hat{\sigma}^{(1)}_{q\bar{q}}}{dtdu}\Big)^{V+S} = \delta(s+t+u-M_W^2)\sigma^{V+S}_{q\bar{q}}(s,t,u,\Delta). \tag{7.11}$$

This contribution to the hadronic cross section takes the same form as those obtained for the Born terms in (7.8) so that $\sigma^{(0)}_{q\bar{q}}$ is replaced by $\sigma^{V+S}_{q\bar{q}}$.

The hard gluon contribution to the hadron cross section is then given by (7.2) where the lower boundary of the integral over x_2 is $x_{2-} = (\Delta + M_W^2 - x_1T)/(x_1S + U)$ (c.f. (7.10)). Integration over x_2 produces terms which are logarithmically dependent on Δ. They originate from terms which behave like $\ln^j(1-x_i)/(1-x_i)\,(j \geq 0)$ and appear in the hard gluon part of $d\sigma$ as well as in the splitting functions $\Gamma_{qq}(x_i)$ (for the latter see (6.15) and (6.16)). Addition of the soft plus virtual and the hard gluon contributions, given in (7.11) and (7.2) respectively, to the hadronic cross section leads to the cancellation of the $\ln^i\Delta$ terms so that the limit $\Delta \to 0$ is defined. Since we do the remaining integrations numerically rather than analytically at this stage we check for stability when Δ is finite and small. Note that if one chooses a specific mass factorization scheme like in (6.17) the $\ln\Delta$ are already cancelled in the $d\sigma^{V+S}_{q\bar{q}}$ and $d\sigma^{H}_{q\bar{q}}$ separately (c.f.(6.19)). In this case the procedure of numerical cancellation is not necessary. The same holds for those processes where there are no gluons in the final state like the one in (2.1).

For processes mentioned in Sec.4 the functions $H(x_1,x_2,Q^2)$ take several forms. Starting with the processes in (2.1) and (4.1) where there are (anti)quarks in the initial state. For W^+ production we have

$$H^{W^+}_{q\bar{q}}(x_1,x_2,Q^2) = u^P(x_1,Q^2)\,\bar{d}^{\bar{P}}(x_2,Q^2) + \bar{d}^P(x_1,Q^2)\,u^{\bar{P}}(x_2,Q^2), \tag{7.12}$$

where the Cabibbo angle has been put equal to zero. For the case of W^- production $u \leftrightarrow \bar{d}$. When we have a gluon and a quark in the initial state H becomes

$$H^{W^+}_{qg}(x_1,x_2,Q^2) = u^P(x_1,Q^2)\,g^{\bar{P}}(x_2,Q^2) + g^P(x_1,Q^2)\,u^{\bar{P}}(x_2,Q^2) \tag{7.13}$$

In the case where there are antiquarks in the initial state $u \to \bar{u}$ and $d \to \bar{d}$. The u, d and g in the above expressions denote the distribution functions of the u, d, s quarks and the gluon respectively.

In order to compute the quantity K defined in (6.18) we have to calculate the total rate for W-production. It is given by

$$\sigma^W_{tot}(S) = \frac{\pi g^{2-}}{12M_W^2}\int_\tau^1 \frac{dx_1}{x_1}\int_{\tau/x_1}^1 \frac{dx_2}{x_2} H_{q\bar{q}}(x_1,x_2,M_W^2)\Delta^{DY}_{q\bar{q}}(x_1,x_2), \tag{7.14}$$

where $\tau = M_W^2/S, H$ is defined in (7.12) and $\Delta^{DY}_{q\bar{q}}$ is the Drell-Yan correction term. If we choose the mass factorization scheme D, with f_{qg} given by (6.13) then $\Delta^{DY}_{q\bar{q}}(x_1,x_2)$ becomes equal to $\delta(\tau/(x_1x_2)-1)$ which leads to a simple expression for σ^W_{tot}. The latter becomes

$$\sigma^W_{tot}(S) = \frac{\sqrt{2}}{3}\pi G_F\tau\int_\tau^1 \frac{dx_1}{x_1} H_{q\bar{q}}(x_1,\frac{\tau}{x_1},M_W^2), \tag{7.15}$$

where we have used $g^2 = 4\sqrt{2}G_FM_W^2$. In the mass factorization scheme D there are no contributions to σ^W_{tot} coming from the qg and $\bar{q}g$ subprocesses.

In order to investigate the stability of the K-factor (6.18) under different mass factorization schemes we have calculated K in scheme C as well as scheme D. Since the parton distribution functions have been determined in deep-inelastic lepton hadron scattering the latter scheme also enables us to make an absolute prediction of the hadronic cross section. Following the discussion below (6.20) the reduced cross section for mass factorization scheme C can be obtained from the reduced cross section in scheme D as follows.

$$\begin{aligned}\Big(s^2\frac{d^2\hat{\sigma}_{ij}}{dtdu}\Big)^C &= \Big(s^2\frac{d^2\hat{\sigma}_{ij}}{dtdu}\Big)^D + \frac{\alpha_s}{2\pi}\int_0^1\frac{dx_1}{x_1}[(f_{ik}(x_1))^D - (f_{ik}(x_1))^C] \\ &\quad\times\hat{s}^2\frac{d^2\sigma^{(0)}_{kj}(x_1s,x_1t,u)}{d\hat{t}du} \\ &\quad+\frac{\alpha_s}{2\pi}\int_0^1\frac{dx_2}{x_2}[(f_{ik}(x_2))^D - (f_{ik}(x_2))^C] \\ &\quad\times\hat{s}^2\frac{d^2\sigma^{(0)}_{kj}(x_2s,t,x_2u)}{dtd\hat{u}}\,.\end{aligned} \tag{7.16}$$

Notice that in the above expression we are not interested in different mass factorization schemes for the final state collinear divergences appearing in the subprocesses $g+q(\bar{q})\to W+\gamma+q(\bar{q})$. Substituting the expressions for $(f_{q\bar{q}})^C$ and $(f_{q\bar{q}})^D$ as given in (6.16) and (6.17) for the subprocess $q+\bar{q}\to W+\gamma+g$ we obtain

$$\begin{aligned}\Big(s^2\frac{d^2\hat{\sigma}_{q\bar{q}}}{dtdu}\Big)^C &= \Big(s^2\frac{d^2\hat{\sigma}_{q\bar{q}}}{dtdu}\Big)^D + \frac{1}{48}\alpha g^2\frac{\alpha_s}{\pi}C_F\Big\{[Q_1(s+t)u+Q_2(M_W^2-u)t]^2 \\ &\quad\times[u^2(s+t)^2+t^2(M_W^2-u)^2+2sM_W^2(M_W^2-u)(s+t)] \\ &\quad\Big[[(s+t)^2+(M_W^2-u)^2][\ln\frac{s+t+u-M_W^2}{s+t}+\frac{3}{4}] \\ &\quad-\frac{1}{4}[9(s+t)+5(M_W^2-u)](s+t+u-M_W^2)\Big] \\ &\quad u^{-1}t^{-1}(s+t)^{-2}(M_W^2-u)^{-2}(tM_W^2+su)^{-2}(s+t+u-M_W^2)^{-1} \\ &\quad\theta(1-\delta_1-\frac{M_W^2-u}{s+t})+(t\leftrightarrow u,Q_1\leftrightarrow Q_2)\Big\} \\ &\quad+\frac{1}{48}\alpha g^2\frac{\alpha_s}{\pi}C_F\delta(s+t+u-M_W^2) \\ &\quad\times\Big\{(Q_1u+Q_2t)^2(t^2+u^2+2sM_W^2)u^{-1}t^{-1} \\ &\quad(s-M_W^2)^{-2}\{\ln^2\delta_1+\frac{3}{2}\ln\delta_1+\frac{1}{2}+4\zeta(2)\}+(t\leftrightarrow u,Q_1\leftrightarrow Q_2,\delta_1\leftrightarrow\delta_2)\Big\}\,.\end{aligned} \tag{7.17}$$

with

$$\delta_1=\frac{\Delta}{s+t}\,,\quad \delta_2=\frac{\Delta}{s+u}\,. \tag{7.18}$$

For the subprocesses $g+q(\bar{q})\to W+\gamma+q(\bar{q})$ we infer $(f_{gq})^C$ and $(f_{gq})^D$ from (6.27)

and (6.28) respectively. The result for $g+q\to W+\gamma+q$ is equal to

$$\begin{aligned}\Big(s^2\frac{d^2\hat{\sigma}_{qg}}{dtdu}\Big)^C &= (s^2\frac{d^2\hat{\sigma}_{qg}}{dtdu})^D + \frac{1}{48}\alpha g^2\frac{\alpha_s}{\pi}T_f[Q_1(M_W^2-t)u+Q_2(s+u)t]^2 \\ &\quad[t^2(s+u)^2+u^2(M_W^2-t)^2+2sM_W^2(M_W^2-t)(s+u)] \\ &\quad\{[(s+u)^2-2(M_W^2-t)(s+u)+2(M_W^2-t)^2](\ln\frac{s+t+u-M_W^2}{s+u}+1) \\ &\quad+\frac{1}{2}(s+t+u-M_W^2)(s+t+5u-5M_W^2)\} \\ &\quad u^{-1}t^{-1}(s+u)^{-3}(M_W^2-t)^{-2}(M_W^2u+st)^{-2}\,.\end{aligned} \tag{7.19}$$

$(d\hat{\sigma}_{q\bar{q}})^C$ can be obtained from $(d\hat{\sigma}_{qg})^C$ by interchanging t with u and Q_1 with Q_2.

Using mass factorization C the total W cross section in (7.15) also changes because we have to add the QCD corrections from the reactions $q+\bar{q}\to W+g$, $g+q\to W+q$ and $g+\bar{q}\to W+\bar{q}$. For the subprocess $q+\bar{q}\to W+g$ we find

$$\begin{aligned}(\sigma^W_{\rm tot})_{q\bar{q}} &= \frac{\sqrt{2}}{3}\pi G_F\tau\Big[\int_\tau^1\frac{dx_1}{x_1}H_{q\bar{q}}(x_1,\frac{\tau}{x_1},M_W^2) \\ &\quad\{1+\frac{\alpha_s(M_W^2)}{\pi}C_F[\ln^2\delta+\frac{3}{2}\ln\delta+\frac{1}{2}+4\zeta(2)]\} \\ &\quad+\frac{\alpha_s(M_W^2)}{\pi}C_F\int_\tau^1\frac{dx_1}{x_1}\int_{\tau(1+\delta)/x_1}^1\frac{dx_2}{x_2}H_{q\bar{q}}(x_1,x_2,M_W^2) \\ &\quad\Big\{\frac{1}{x_1x_2}\frac{x_1^2x_2^2+\tau^2}{x_1x_2-\tau}\ln(1-\frac{\tau}{x_1x_2})-3-2\frac{\tau}{x_1x_2}+\frac{3}{2}\frac{x_1x_2}{x_1x_2-\tau}\Big\}\Big]\,.\end{aligned} \tag{7.20}$$

For the subprocesses $g+q(\bar{q})\to W+q(\bar{q})$ we find

$$\begin{aligned}(\sigma^W_{\rm tot})_{qg} &= \frac{\sqrt{2}}{3}\pi G_F\tau\Big[\frac{\alpha_s(M_W^2)}{2\pi}T_f\int_\tau^1\frac{dx_1}{x_1}\int_{\tau/x_1}^1\frac{dx_2}{x_2}H_{qg}(x_1,x_2,M_W^2) \\ &\quad\Big\{[\frac{\tau^2}{x_1^2x_2^2}+\big(1-\frac{\tau}{x_1x_2}\big)^2]\ln(1-\frac{\tau}{x_1x_2})+\frac{3}{2}-5\frac{\tau}{x_1x_2}+\frac{9}{2}\frac{\tau^2}{x_1^2x_2^2}\Big\}\Big]\,,\end{aligned} \tag{7.21}$$

with the same expression for $(\sigma^W_{\rm tot})_{\bar{q}g}$. The final expression for the K-factor is therefore

$$K=\frac{d\sigma_{q\bar{q}}+d\sigma_{qg}+d\sigma_{q\bar{g}}}{(\sigma^W_{\rm tot})_{q\bar{q}}+(\sigma^W_{\rm tot})_{qg}+(\sigma^W_{\rm tot})_{\bar{q}g}} \tag{7.22}$$

8 Results

Now that all the divergences associated with the first order QCD corections have been extracted from the squares of the matrix elements, the total cross sections and the various differential distributions of experimental interest can be computed. We assume that the W-boson is identified by one of its decay modes, and has the mass $M_W = 82.0\,\mathrm{GeV/c^2}$. The identification of the photon is essential in establishing the existence of the radiative reaction $p+\bar{p} \to W+\gamma+X$ so cuts have to be imposed on the energy and angle of the photon to take experimental acceptances into account.

Due to the exchange of massless quarks in the t and u channels the cross sections for the Born (2.5) as well as the soft plus virtual (6.19) and the hard (6.20), (6.33), (7.17), (7.19) processes diverge when t and u tend to zero. Since t and u are small when the photon is produced in the forward and backward directions, the photon angular distributions are peaked in these regions. Also the parton-parton contributions to the cross sections for $p+\bar{p} \to W+\gamma+X$ are infrared divergent in the energy of the photon, so we have to impose a minimum energy cut. Since the invariant mass $S_4 = S+T+U$ of the W-gluon pair tends to its maximum as the photon energy goes to zero, we expect to see a typical bremsstrahlung peaking of the cross section near $S_4 = S_{4,\max} = S$, in which limit T and U also become zero. We avoid these regions by appropriate cuts on the energy and angle of the photon. At present we have simply chosen reasonable values for these cuts, namely $E_\gamma > 10$ GeV, and $0.1\,\mathrm{rad} < \theta < (\pi - 0.1)\,\mathrm{rad}$ where, for parton-parton cross sections E_γ is the energy of the photon in the center-of-mass system of the partons and θ is the angle of the photon in the same system with respect to the beam axis, while for $p\bar{p}$ cross sections these quantities are defined with respect to the center-of-mass frame of the $p\bar{p}$. The other apparent divergence at threshold $s \to M_W^2$ disappears when all the contributions to the cross section are added together, but is anyway avoided by the minimum energy cut for the photon.

The values of the cross sections are clearly sensitive to the specific choices for these cuts. However, remember that our aim here is to investigate the modifications of the differential cross section in the neighbourhood of the radiation zero so we are more interested in studying those effects which change the photon differential cross section at large angles than the study of cuts which change the angular distribution near its endpoints. Even after the implementation of the cuts mentioned above there are several maxima in the differential distributions, so care has to be taken in the numerical analysis. We used the program Vegas [31] for the integrations, together with mappings to smooth out the remaining peaks.

Another remark we make here regards the apparent divergence in the variable s_4 due to the factor $(s_4 - M_W^2)^{-1}$, appearing in (6.20). As we mentioned previously there is no singularity at this point in the mass factorization scheme D since the soft plus virtual and the hard contributions are individually finite. The apparent divergence in the mass factorization term in (6.20) is cancelled by corresponding divergences in terms in the integrals listed in Appendix B. We have analytically checked this cancellation for the $q\bar{q}$ parton-parton process. The hard contribution

to $p+\bar{p} \to W+\gamma+X$ can therefore be integrated over s_4 down to its exact minimum. We tested this numerically, using a very small cut-off on the photon energy. In mass factorization scheme C we checked the cancellation of the terms in $\ln\Delta$ numerically.

Now we should mention our choice of structure functions. We computed all our results for the $p\bar{p}$ reaction cross sections with both the Duke and Owens [32] structure functions for valence quarks and gluons and the recent set provided by Diemoz et al[33](DFLM). Since there were no significant differences in the results we will simply report those from the latter set here. Other fits to the structure functions are listed in Ref.34. We have always taken $\cos\theta_C$, the cosine of the Cabibbo angle, equal to unity, and ignored the strange quarks since $\sin\theta_C$ is small.

For the running coupling constant we have chosen

$$\alpha_s(Q^2) = \frac{12\pi}{(33-2n_f)\ln(Q^2/\Lambda^2)} \tag{8.1}$$

with $\Lambda = 0.2\,\mathrm{GeV}$, $Q^2 = s$ and $n_f = 6$.

To make contact with previous results in Refs 3 and 13 we first plot the photon differential cross section $d\sigma/d\cos\theta_\gamma$ in Fig. 6 for the parton-parton Born reaction $u+\bar{d} \to W^+ + \gamma$ at $\sqrt{s} = 95\,\mathrm{GeV}$, with $0.1 < \cos\theta < \pi - 0.1$. where θ the angle between the photon and the direction of the u quark. The radiation zero in (2.5) occurs at $\cos\theta = (Q_1+Q_2)/(Q_2-Q_1) = -1/3$.

The Born contribution to the hadronic cross sections $d\sigma/d\cos\theta_\gamma$ and $d\sigma/dE_\gamma$ for the reaction $p+\bar{p} \to W^+ + \gamma + X$ are given for $\sqrt{S} = 0.63$ TeV (CERN SPS) in Figs. 7 and 8 respectively. Here θ stands for the angle of the photon with respect to the incoming proton. In these and our subsequent figures we have used the DLFM structure functions [33] only. The same cross sections are also plotted for $\sqrt{S} = 1.8$ TeV (Fermilab Tevatron) in Figs 9 and 10. In order to present the K factor for the differential distributions in (6.18) we have divided $d\sigma/d\cos\theta_\gamma$ and $d\sigma/dE_\gamma$ by the Born contribution to the DY cross section $\sigma_{\mathrm{tot}}^{W^+}(S)$ (7.15). The latter has the value 2.17 nb at $\sqrt{S} = 0.63$ TeV and 7.78 nb at $\sqrt{S} = 1.8$ TeV . Due to the convolution of the parton cross section with the structure functions the radiation zero in the Born parton cross section at $\cos\theta = -1/3$ is now modified into a dip. The integrated results for $E_\gamma > 10\,\mathrm{GeV}$ and $0.1 < \theta < \pi - 0.1$ yield $K = 2.67 \times 10^{-3}$ and $K = 3.06 \times 10^{-3}$ respectively (see 6.18). This implies that the total Born cross section for the reaction $p+\bar{p} \to W^+ + \gamma + X$ becomes 5.80 pb for $\sqrt{S} = 0.63$ TeV whereas for $\sqrt{S} = 1.8$ TeV it equals 23.8 pb.

Let us now present the results when we include QCD corrections calculated according to mass factorization scheme D. The order α_s corrections to the parton differential cross sections are extremely small and fluctuate in sign both for the soft plus virtual as well as hard gluon contributions. This sign fluctuation does not happen with pure QED radiative corrections where hard bremsstrahlung cross sections are positive definite. The reason is easy to find. The subtraction of the collinear singularities from the hard parton cross section sometimes results in negative contributions in regions of phase space. In such cases we have always computed the positive and negative parts separately. However this means that it is not informa-

fill in the dip (Fig. 7) more than in the case of the parton cross section in Fig 6. However the shape of the distributions in Figs 7 and 8 is not that drasically changed that it could be seen experimentlly. The same can be observed for larger CM energies. In Figs 9 and 10 we have plotted the corresponding distributions at $\sqrt{S} = 1.8$ TeV, where we have divided by the DY cross section in scheme C, which has the value 10.2 nb. Here the agreement between the exact and the approximate order α_s correction is not quite so good. Therefore we show both curves in Figs 9 and 10. The approximate result is uniformly larger than the exact one due to the fact that at Fermilab collider energies the hard gluon part of the $q\bar{q}$ cannot be neglected anymore. The same holds for the negative contributions from the qg and $\bar{q}g$ channels. Since the size of the negative part of the latter processes is larger than the positive contribution due to hard gluon radiation the exact curve lies below the approximate one. This feature is also shown by the DY process. The reason that at large energies the qg and $\bar{q}g$ processes become more important can be traced back to the input gluon distribution function $g(x)$ (7.13) which becomes very big when x is very small. Notice that near the parton threshold $s = x_1 x_2 S \geq M_W^2$ the fraction x of the gluon decreases when S grows. Using the exact answers the integrated results for (6.18) yield $K = 3.1 \times 10^{-3}$ ($\sqrt{S} = 0.63$ TeV) and $K = 3.0 \times 10^{-3}$ ($\sqrt{S} = 1.8$ TeV). These numbers are only slightly different from those obtained for the Born approximation. From Figs. 7-10 we also infer that the K factor for the differential distributions as defined in (6.18) is hardly changed by the order α_s corrections. Since the input parton distribution functions are measured in deep inelastic lepton hadron scattering in mass factorization scheme C we are also able to give absolute predictions for the differential and total cross sections mentioned above. For $\sqrt{S} = 0.63$ TeV the total cross section for $p + \bar{p} \to W^{\mp} + \gamma + X$ equals 9.49 pb whereas for $\sqrt{S} = 1.8$ TeV it equals 30.3 pb.

Summarizing our findings we conclude the following. A complete order α_s QCD correction has been computed for the reaction $p + \bar{p} \to W + \gamma + X$. This includes a complete evaluation of the virtual soft and hard gluon corrections to the Born process $q + \bar{q} \to W + \gamma$. In addition the contributions of the qg and $\bar{q}g$ channels have been taken into account. The QCD corrections to the parton Born cross section do not modify the shape of the photon differential cross section. The corrections to the position of the radiation zero are unnoticeable. In the hadronic $p\bar{p}$ reaction the zero is changed into a dip due to the convolution of the parton cross section with the input parton distribution functions. This already shows up at the Born level. The order α_s corrections fill in the dip even more but they are so small that they will not be experimentally observable. Further we notice that the shape of the distributions are not appreciably altered by the QCD corrections which is revealed by the K factor in (6.18). There is a change in the normalization only which is the same effect as seen for the DY process. Therefore the integrated K factor does not change under QCD corrections. This feature is a consequence of the fact that these corrections are dominated by the threshold region. This explains why there is no appreciable difference between mass factorization scheme C and D. The latter is due to the fact that the difference between C and D can be

traced back to a normalization effect which cancels in the computation of K. The production rates given by the order α_s corrected cross sections are slightly larger than those obtained from the Born approximation. Our findings above are different from those observed by us previously in Ref. 10 where the QCD corrections to the decay $W \to q + \bar{q} + \gamma$ were studied. In the latter case the shape of the distributions were considrably modified by the final state gluon corrections in particular at the edges of the $d\sigma/d\cos\theta$ plot. Fortunately the W will be mainly observed via its leptonic decay channels so that we have not to worry about these corrections. The QED corrections to the leptonic channel are already available in the literature. Since the initial state QCD corrections to the shape of the distributions are small it is probably possible to measure the gyromagnetic ratio κ of the W boson via the process $p + \bar{p} \to W + \gamma + X$. However it will require a significant increase in number of W boson events at CERN and Fermilab before these radiative processes can be studied experimentally.

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tive to give a listing of the individual contributions to the cross section. One may think that this problem can be circumvented in the case of the soft plus virtual contributions to the differential cross section by always adding them to those of the Born cross section, which has the same kinematics, thereby producing a positive result. However this is not true in our case since the Born contribution is extremely small in the neighbourhood of the radiation zero so the soft plus virtual part makes the total sum negative near that point. The only reasonable way to present the results is to take the sum of all contributions i.e. hard, soft plus virtual and Born. Even in this case one sometimes obtains negative differential cross sections near the radiation zero. Note that it is erroneous to conclude that perturbation theory breaks down since these differential cross sections are are not physical observables. It is therefore important to note that these negative pieces vanish after convolution with the parton distribution functions. Finally we observe that the (anti) quark gluon initial state processes give very small negative corrections. Since in mass factorization scheme D the corrections are very small and only slightly change the lowest order differential cross sections we have not plotted them. To give an impression of their size we have presented the total parton parton cross sections with the cuts mentioned above in table 1.

For the $p\bar{p}$ reaction using mass factorization D we find essentially no change in the distributions and K values given above for the Born reaction. It is not surprising that mass factorization scheme D leads to such small corrections because of the similarity between the reaction $u + \bar{d} \to W^+ + \gamma + X$ and the DY process $u + \bar{d} \to W^+ + X$. In the latter reaction scheme D leads to a correction term which is equal to zero. Notice that in order to make absolute predictions in mass factorization scheme D we need the input parton distribution functions measured in the DY process.

The results obtained in mass factorization scheme C differ appreciably from those calculated in D. This holds for the parton as well as for the hadronic cross sections as we will show below. With the cuts given above we have computed the order α_s correction to the total parton cross section for several values of the parton CM energies. The results are presented in table 1. Notice that in the case of scheme C the expressions for the virtual plus soft and the hard gluon parts involve the parameter Δ (6.11). We checked that the sum of all these contributions becomes independent of Δ when $\Delta < 0.01\text{GeV}^2$. Therefore we have chosen this value for the computation of the numbers in table 1. From the table we infer that the corrections become very large when $\sqrt{s}$ approaches the threshold energy. For $\sqrt{s} > 105$ GeV the corrections are small and become of the same order of magnitude as those found in mass factorization scheme D. These large corrections can only be attributed to the $q + \bar{q} \to W^+ + \gamma + g$ subprocess since the contributions from the qg and $q\bar{q}$ processes are small near threshold. The same phenomenon has also been observed for the DY reaction. Since these large corrections are caused by soft plus virtual gluons we can approximate the correction term in scheme C by the following

formula which can be derived from (7.17) in the limit $s_4 \to M_W^2$.

$$\left(s^2\frac{d^2\hat{\sigma}_{q\bar{q}}}{dtdu}\right)^C = \frac{C_F\alpha_s}{\pi}\{\ln\frac{(s+t+u-M_W^2)^2}{(s+t)(s+u)}+\frac{3}{2}\}\frac{1}{(s+t+u-M_W^2)}$$
$$+\delta(s+t+u-M_W^2)\{\frac{1}{2}\ln^2\left(\frac{\Delta}{s+t}\right)+\frac{1}{2}\ln^2\left(\frac{\Delta}{s+u}\right)$$
$$+\frac{3}{4}\ln\left(\frac{\Delta^2}{(s+t)(s+u)}\right)+\frac{1}{2}+4\zeta(2)\}] \tag{8.2}$$

As can be seen in table 1, (8.2) gives a good approximation to the total parton cross section in the vicinity of the threshold $\sqrt{s} < 100$ GeV. This implies that the hadronic cross section also gets its main contribution from the threshold region due to the rapid decrease in the parton cross sections with increasing s. Therefore we have given the order α_s corrected parton cross section $d\hat{\sigma}/d\cos\theta_\gamma$ in Fig. 9, for a representative value of the parton CM energy i.e. $\sqrt{s} = 95$ GeV. For the same value of $\sqrt{s}$ we have also calculated the results from the approximate formula (8.2). In all of the angular regions the agreement with the exact distribution is excellent which means that the soft and virtual gluons dominate the QCD correction terms. Further we infer from Fig 6 that the shape of the Born cross section is not altered by the order α_s correction. This can be attributed to the following effects. Since $\sqrt{s} = 95$ GeV is very close to threshold the soft and virtual gluon parts dominate the process and they have the same zero as the Born cross section (see eq. 6.19). In other words they are positive. The hard gluon contribution is negative as well as the dominant part of the contribution from the qg and $\bar{q}g$ channels. Hence near the zero the latter two terms actually produce a very small negative contribution which is too small to plot. The radiation zero present in the Born approximation is therefore changed by the hard gluon radiation and the contributions of the qg and $\bar{q}g$ subprocesses but not in the way one might at first expect. These effects persist even at higher energies (e.g. $\sqrt{s} = 200$ GeV) so that in the region where the latter processes become more important the net correction to the zero is still rather small. Hence it is completely wrong to assume that a positive hard gluon contribution fills in the dip. We have already made remarks above about similar effects in mass factorization scheme D. For $\sqrt{s} = 95$ GeV the figure for $d\sigma/dE$ could not be presented due to the limited amount of phase space available for the photon. It is very close to a delta function and therefore appears as a very narrow spike.

In Figs 7 and 8 we have plotted the order α_s corrected hadronic cross sections $\left(\sigma_{\text{tot}}^{W+}\right)^{-1} d\sigma/d\cos\theta_\gamma$ and $\left(\sigma_{\text{tot}}^{W+}\right)^{-1} d\sigma/dE_\gamma$ for the reaction $p + \bar{p} \to W^+ + \gamma + X$ at $\sqrt{S} = 0.63$ TeV using the DFLM structure functions. The distributions as well as σ_{tot}^{W+} are computed in mass factorization scheme C with the standard cuts mentioned above and we have divided by the DY cross section in scheme C, which has the value 3.01 nb at $\sqrt{S} = 0.63$ TeV. The results from the approximation in (8.2) are identical to those obtained from the exact order α_s correction. Due to the convolution of the parton cross section with the parton distribution functions the order α_s corrections

Appendix A

In this Appendix we outline the kinematics for the reactions discussed in the text. The two to three body process in (4.1) is described in the $C.M.$ frame of the recoiling gluon and W-boson[25][30]. In n-dimensions the momenta are given by

$$\begin{aligned} p_1 &= (E_1, 0, ..., 0, 0, E_1) && \text{(A.1)} \\ p_2 &= (E_2, 0, ..., 0, \omega \sin\psi, \omega\cos\psi - E_1) && \text{(A.2)} \\ k &= (\omega, 0, ..., 0, \omega\sin\psi, \omega\cos\psi) && \text{(A.3)} \\ k' &= (\omega', ..., \omega' \sin\theta_1 \sin\theta_2, \omega' \sin\theta_1 \cos\theta_2, \omega'\cos\theta_1) && \text{(A.4)} \\ q &= (\omega_W, ..., -\omega' \sin\theta_1 \sin\theta_2, -\omega' \sin\theta_1 \cos\theta_2, -\omega'\cos\theta_1), && \text{(A.5)} \end{aligned}$$

where in (A.1)-(A.5) the dots stand for $n-3$ equal and opposite angular components.

From four momentum conservation and the on-mass-shell constraints one can derive the following identities

$$\begin{aligned} E_1 &= \frac{s+t}{2\sqrt{s_4}}, \quad E_2 = \frac{s+u}{2\sqrt{s_4}}, \quad \cos\psi = \frac{us - s_4 t}{(t+u)(t+s)}, \\ \omega &= -\frac{t+u}{2\sqrt{s_4}}, \quad \omega' = \frac{s_4 - M_W^2}{2\sqrt{s_4}}, \quad \omega_W = \frac{s_4 + M_W^2}{2\sqrt{s_4}}, \end{aligned} \tag{A.6}$$

where s, t, u and s_4 are defined in (4.2). Further we have the relation

$$s + t + u = s_4. \tag{A.7}$$

The cross section is given by

$$\begin{aligned} \sigma &= \frac{1}{3}\Big(\frac{1}{2}\Big)^2 \frac{\mu^{-2\epsilon}}{2s} \int \frac{d^n k}{(2\pi)^{n-1}} \int \frac{d^n k'}{(2\pi)^{n-1}} \int \frac{d^n q}{(2\pi)^{n-1}} \delta^+(k^2)\delta^+(k'^2) \\ &\quad \times \delta^+(q^2 - M_W^2)(2\pi)^n \delta^{(n)}(p_1 + p_2 - k - k' - q)|M^R|^2, \end{aligned} \tag{A.8}$$

where M^R is the matrix element of the process given in (4.1), and $\epsilon = n - 4$. The factor $(1/2)^2$ is from the average over the initial quark spins, and the $1/3$ is from the average over the colour degree of freedom. The integral can be rewritten as follows

$$\begin{aligned} \sigma &= \frac{1}{3}\Big(\frac{1}{2}\Big)^2 \frac{1}{2s} \frac{\mu^{-2\epsilon}}{(2\pi)^{2n-3}} \int d^n k d^n p \delta^+(k^2)\delta^{(n)}(p_1 + p_2 - k - p) \\ &\quad \times \int d^n k' d^n q \delta^+(k'^2)\delta^+(q^2 - M_W^2)\delta^{(n)}(p - k' - q)|M^R|^2. \end{aligned} \tag{A.9}$$

The second factor in (A.9) is the standard two body phase space integral. Its evaluation in the C.M. frame of the gluon and W-boson yields,

$$\begin{aligned} &\int d^n k' d^n q \delta(k'^2)\delta^+(q^2 - M_W^2)\delta^{(n)}(p - k' - q)|M^R|^2 = \\ &\frac{1}{4}\pi^{n/2-2} \frac{\Gamma(n/2-1)}{\Gamma(n-3)} \frac{(p^2 - M_W^2)^{n-3}}{(p^2)^{n/2-1}} \int_0^\pi d\theta_1 \sin^{n-3}\theta_1 \int_0^\pi d\theta_2 \sin^{n-4}\theta_2 |M^R|^2. \end{aligned} \tag{A.10}$$

After integration over the n-dimensional vector p, we can express the cross section in variables chosen in the C.M. frame of the photon and the W-gluon pair. In this frame we have the parametrization

$$\begin{aligned} p_1 &= \frac{1}{2}\sqrt{s}(1, 0, ..., 0, 1) \\ p_2 &= \frac{1}{2}\sqrt{s}(1, 0, ..., 0, -1) \\ k &= \omega(1, 0, ..., 0, \sin\chi, \cos\chi). \end{aligned} \tag{A.11}$$

so the cross section is

$$\begin{aligned} \sigma &= \frac{1}{3}\Big(\frac{1}{2}\Big)^2 \frac{1}{s} \frac{S_\epsilon^2 \mu^{-2\epsilon}}{\Gamma(n-3)} \int d\omega \int_0^\pi d\chi \omega^{n-3} \sin^{n-3}\chi \\ &\quad \times \frac{(s_4 - M_W^2)^{n-3}}{s_4^{n/2-1}} \int_0^\pi d\theta_1 \sin^{n-3}\theta_1 \int_0^\pi d\theta_2 \sin^{n-4}\theta_2 |M^R|^2, \end{aligned} \tag{A.12}$$

where we have integrated over all angles of the photon momentum which do not appear in $|M^R|^2$. After changing the integration variables ω and χ to t and u, using

$$\omega = -\frac{t+u}{2\sqrt{s}}, \qquad \cos\chi = \frac{u-t}{u+t}, \tag{A.13}$$

we find

$$\begin{aligned} s^2 \frac{d^2\sigma_{q\bar{q}}}{dtdu} &= \frac{1}{3}\Big(\frac{1}{2}\Big)^3 \frac{S_\epsilon^2 \mu^{-\epsilon}}{\Gamma(n-3)} \Big(\frac{ut}{s\mu^2}\Big)^{n/2-2} \frac{(s_4 - M_W^2)^{n-3}}{s_4^{n/2-1}} \\ &\quad \times \int_0^\pi d\theta_1 \sin^{n-3}\theta_1 \int_0^\pi d\theta_2 \sin^{n-4}\theta_2 |M^R|^2. \end{aligned} \tag{A.14}$$

which is the result used in Sec.4.

The kinematics for the reactions $q + g \to W + \gamma + q$ and $\bar{q} + g \to W + \gamma + \bar{q}$ are similar to those for the reaction $q + \bar{q} \to W + \gamma + g$. In these cases the factor of $(1/2)^2$ appears for the average over (anti-) quark-gluon spins. We therefore use the formulae derived above in the c.m. frames of the Wq and $W\bar{q}$ to determine $s^2 d^2\sigma_{qg}/dtdu$ and $s^2 d^2\sigma_{\bar{q}g}/dtdu$ respectively.

Appendix B

Here we give the angular integrals of the terms $(s')^k(s'')^l$ which arise from the partial fractioning of the square of matrix element $|M^R|^2$ in (4.3). The general expression for the angular integral is given in (4.8). The specific 4-dimensional integrals for $a^2 \neq b^2$ and $A^2 \neq B^2 + C^2$ are listed first[20].

$$I_4^{(0,0)} = 2\pi\,, \tag{B.1}$$

$$I_4^{(-1,0)} = 2\pi a\,, \tag{B.2}$$

$$I_4^{(0,-1)} = 2\pi A\,, \tag{B.3}$$

$$I_4^{(1,0)} = \frac{\pi}{b}\ln\frac{a+b}{a-b}\,, \tag{B.4}$$

$$I_4^{(0,1)} = \frac{\pi}{\sqrt{B^2+C^2}}\ln\left(\frac{A+\sqrt{B^2+C^2}}{A-\sqrt{B^2+C^2}}\right)\,, \tag{B.5}$$

$$I_4^{(1,1)} = \frac{\pi}{\sqrt{X}}\ln\left(\frac{aA-bB+\sqrt{X}}{aA-bB-\sqrt{X}}\right)\,, \tag{B.6}$$

with $X = (aA-bB)^2 - (A^2-B^2-C^2)(a^2-b^2)$,

$$I_4^{(-1,-1)} = 2\pi\,(aA+\frac{1}{3}bB)\,, \tag{B.7}$$

$$I_4^{(-1,1)} = \pi\left[\frac{2bB}{B^2+C^2} + \frac{a(B^2+C^2)-bAB}{(B^2+C^2)^{3/2}}\ln\left(\frac{A+\sqrt{B^2+C^2}}{A-\sqrt{B^2+C^2}}\right)\right]\,, \tag{B.8}$$

$$I_4^{(0,2)} = 2\pi\,\frac{1}{A^2-B^2-C^2}\,, \tag{B.9}$$

$$I_4^{(-2,0)} = 2\pi\,(a^2+\frac{1}{3}b^2)\,, \tag{B.10}$$

$$I_4^{(0,-2)} = 2\pi\,(A^2+\frac{1}{3}(B^2+C^2))\,, \tag{B.11}$$

$$I_4^{(-2,1)} = \pi\Big[\frac{4abB}{B^2+C^2} + \frac{b^2A(C^2-2B^2)}{(B^2+C^2)^2} + \Big\{\frac{(a(B^2+C^2)-bAB)^2}{(B^2+C^2)^{5/2}} - \frac{b^2C^2(A^2-B^2-C^2)}{2(B^2+C^2)^{5/2}}\Big\} \times \ln\Big(\frac{A+\sqrt{B^2+C^2}}{A-\sqrt{B^2+C^2}}\Big)\Big]\,, \tag{B.12}$$

$$I_4^{(-1,2)} = \pi\left[\frac{2[a(B^2+C^2)-bAB]}{(B^2+C^2)(A^2-B^2-C^2)} + \frac{bB}{(B^2+C^2)^{3/2}}\ln\left(\frac{A+\sqrt{B^2+C^2}}{A-\sqrt{B^2+C^2}}\right)\right]\,. \tag{B.13}$$

In n-dimensions [18][20] we need the following integrals, which we classify into groups where $b = -a, A^2 = B^2 + C^2$, or $b = -a, A^2 \neq B^2 + C^2$ or $a^2 \neq b^2, A^2 = B^2 + C^2$ respectively.

Group 1. $b = -a$ and $A^2 = B^2 + C^2$.

$$\hat{I}_n^{(1,0)} = 2\pi\,\frac{1}{a}\,\frac{1}{n-4}\,. \tag{B.14}$$

$$\hat{I}_n^{(1,1)} = 2\pi\,\frac{1}{aA}\,\frac{1}{n-4}\;\mathrm{F}_{1,2}\left(1,1,\frac{1}{2}n-1,\frac{A-B}{2A}\right)\,, = 2\pi\,\frac{1}{aA}\,\frac{1}{n-4}\left(\frac{A+B}{2A}\right)^{n/2-3}\left[1+\frac{1}{4}(n-4)^2\mathrm{Li}_2\left(\frac{A-B}{2A}\right) + \mathrm{O}((n-4)^3)\right]\,, \tag{B.15}$$

$$\hat{I}_n^{(-1,1)} = 2\pi\,\frac{a}{A}\,\frac{n-2}{(n-4)(n-3)}\mathrm{F}_{1,2}\left(-1,1,\frac{1}{2}n-1,\frac{A-B}{2A}\right) = \pi\,\frac{a(A+B)}{A^2}\left[\frac{2}{n-4} - \frac{2B}{A+B} + \mathrm{O}(n-4)\right]\,, \tag{B.16}$$

$$\hat{I}_n^{(-2,1)} = 2\pi\,\frac{a^2}{A}\,\frac{n}{(n-4)(n-3)}\mathrm{F}_{1,2}\left(-2,1,\frac{1}{2}n-1,\frac{A-B}{2A}\right) = \pi\,\frac{a^2(A+B)^2}{A^3}\left[\frac{2}{n-4} + \frac{A^2-4AB-3B^2}{(A+B)^2} + \mathrm{O}(n-4)\right]\,. \tag{B.17}$$

Group 2. $b = -a$ and $A^2 \neq B^2 + C^2$.

$$\tilde{I}_n^{(1,-1)} = \pi\,\frac{A+B}{a}\left[\frac{2}{n-4} - \frac{2B}{A+B} + \mathrm{O}(n-4)\right]\,, \tag{B.18}$$

$$\bar{I}_n^{(1,-2)} = \pi\frac{(A+B)^2}{a}\Big[\frac{2}{n-4}+\frac{C^2-4AB-2B^2}{(A+B)^2}+\mathrm{O}(n-4)\Big], \tag{B.19}$$

$$\begin{aligned}\bar{I}_n^{(1,1)} = {}& \pi\frac{1}{a(A+B)}\Big[\frac{2}{n-4}+\ln\Big(\frac{(A+B)^2}{A^2-B^2-C^2}\Big)\\ &+\frac{1}{2}(n-4)\Big\{\ln^2\Big(\frac{A-\sqrt{B^2+C^2}}{A+B}\Big)-\frac{1}{2}\ln^2\Big(\frac{A+\sqrt{B^2+C^2}}{A-\sqrt{B^2+C^2}}\Big)\\ &+2\mathrm{Li}_2\Big(-\frac{B+\sqrt{B^2+C^2}}{A-\sqrt{B^2+C^2}}\Big)-2\mathrm{Li}_2\Big(\frac{B-\sqrt{B^2+C^2}}{A+B}\Big)\Big\}\\ &+\mathrm{O}((n-4)^2)\Big],\end{aligned} \tag{B.20}$$

$$\begin{aligned}\bar{I}_n^{(1,2)} = {}& \pi\frac{1}{a(A+B)^2}\Big[\frac{2}{n-4}+\ln\Big(\frac{(A+B)^2}{A^2-B^2-C^2}\Big)+\frac{2(B^2+C^2+AB)}{A^2-B^2-C^2}\\ &+\frac{(n-4)}{2}\Big\{\ln^2\Big(\frac{A-\sqrt{B^2+C^2}}{A+B}\Big)-\frac{1}{2}\ln^2\Big(\frac{A+\sqrt{B^2+C^2}}{A-\sqrt{B^2+C^2}}\Big)\\ &+2\mathrm{Li}_2\Big(-\frac{B+\sqrt{B^2+C^2}}{A-\sqrt{B^2+C^2}}\Big)-2\mathrm{Li}_2\Big(\frac{B-\sqrt{B^2+C^2}}{A+B}\Big)\\ &-2\frac{(A+B)\sqrt{B^2+C^2}}{A^2-B^2-C^2}\ln\Big(\frac{A+\sqrt{B^2+C^2}}{A-\sqrt{B^2+C^2}}\Big)-2\ln\Big(\frac{(A+B)^2}{A^2-B^2-C^2}\Big)\Big\}\\ &+\mathrm{O}((n-4)^2)\Big].\end{aligned} \tag{B.21}$$

Group 3. $a^2 \neq b^2$ and $A^2 = B^2 + C^2$.

$$\bar{I}_n^{(1,1)} = \pi\frac{1}{aA-bB}\Big[\frac{2}{n-4}+\ln\frac{(aA-bB)^2}{(a^2-b^2)A^2}+\mathrm{O}(n-4)\Big], \tag{B.22}$$

$$\begin{aligned}\bar{I}_n^{(2,1)} = {}& \pi\frac{A}{(aA-bB)^2}\Big[\frac{2}{n-4}+\ln\frac{(aA-bB)^2}{(a^2-b^2)A^2}\\ &+\frac{2b(bA-aB)}{A(a^2-b^2)}+\mathrm{O}(n-4)\Big].\end{aligned} \tag{B.23}$$

From the partial fractioning of the two to three body matrix element in (4.5) we obtain 32 independent integrals. However, since the parametrization of the incoming momenta can be interchanged using (4.6) there exists a $t \leftrightarrow u$ symmetry. This implies that the number of independent functions is reduced. It appears that there are only 19 independent types. These 19 integrals belong to 7 different classes which are characterized by the dependence of the functions a, b, A, B and C on the variables s, t, u and M_W^2. Since many integrals can be obtained from the other ones by interchanging t and u we drop the explicit dependence on s and M_W^2 in the following functions. Further we denote the angular measure by

$$d\Omega_n = d\theta_1 \sin^{n-3}\theta_1 d\theta_2 \sin^{n-4}\theta_2 . \tag{B.24}$$

The 7 classes are listed below.
I.

$$a(t,u) = \frac{(M_W^2-s-t-u)(s+t)}{2(s+t+u)}, \qquad b(t,u) = -a(t,u), \tag{B.25}$$

$$A(t,u) = \frac{2M_W^2 t-(s+t+u-M_W^2)(s+u)}{2(s+t+u)}, \tag{B.26}$$

$$B(t,u) = \frac{(s+t+u-M_W^2)(s(s+t+u)-ut)}{2(s+t)(s+t+u)}, \tag{B.27}$$

$$C(t,u) = (s+t+u-M_W^2)\left(\frac{stu}{(s+t)^2(s+t+u)}\right)^{1/2}, \tag{B.28}$$

$$R_0 = \int d\Omega_4\, 1 = I_4^{(0,0)}, \tag{B.29}$$

$$R_1 = \int d\Omega_4\, t' = I_4^{(-1,0)}(a(t,u)), \tag{B.30}$$

$$R_2 = \int d\Omega_4\, t'^2 = I_4^{(-2,0)}(a(t,u), b(t,u)) \tag{B.31}$$

$$R_3 = \int d\Omega_4\, u_7 = I_4^{(0,-1)}(A(t,u)), \tag{B.32}$$

$$R_4 = \int d\Omega_4\, u_7^2 = I_4^{(0,-2)}(A(t,u), B(t,u), C(t,u)), \tag{B.33}$$

$$R_5 = \int d\Omega_4\, t' u_7 = I_4^{(-1,-1)}(a(t,u), b(t,u), A(t,u), B(t,u)), \tag{B.34}$$

$$R_6 = \int d\Omega_n\, t'^{-1} = \bar{I}_n^{(1,0)}(a(t,u)), \tag{B.35}$$

$$R_7 = \int d\Omega_4\, u_7^{-1} = I_4^{(0,1)}(A(t,u), B(t,u), C(t,u)), \tag{B.36}$$

$$R_8 = \int d\Omega_4\, u_7^{-2} = I_4^{(0,2)}(A(t,u), B(t,u), C(t,u)), \tag{B.37}$$

$$R_9 = \int d\Omega_n\, t'^{-1}u_7^{-1} = \bar{I}_n^{(1,1)}(a(t,u), A(t,u), B(t,u), C(t,u)), \tag{B.38}$$

$$R_{10} = \int d\Omega_n\, t'^{-1}u_7 = \bar{I}_n^{(1,-1)}(a(t,u), A(t,u), B(t,u), C(t,u)), \tag{B.39}$$

$$R_{11} = \int d\Omega_n\, t'^{-1}u_7^2 = \bar{I}_n^{(1,-2)}(a(t,u), A(t,u), B(t,u), C(t,u)), \tag{B.40}$$

$$R_{12} = \int d\Omega_n\, t'^{-1}u_7^{-2} = \bar{I}_n^{(1,2)}(a(t,u), A(t,u), B(t,u), C(t,u)), \tag{B.41}$$

$$R_{13} = \int d\Omega_4\, t' u_7^{-1} = I_4^{(-1,1)}(a(t,u), b(t,u), A(t,u), B(t,u), C(t,u)), \tag{B.42}$$

$$R_{14} = \int d\Omega_4\, t' u_7^{-2} = I_4^{(-1,2)}(a(t,u), b(t,u), A(t,u), B(t,u), C(t,u)), \tag{B.43}$$

The integrals over $u'(R_{15}), u'^2(R_{16}), u_6(R_{17}), u_6^2(R_{18}), u'u_6(R_{19})$, $u'^{-1}(R_{20}), u_6^{-1}(R_{21}), u_6^{-2}(R_{22}), u'^{-1}u_6^{-1}(R_{23}), u'^{-1}u_6(R_{24})$, $u'^{-1}u_6^2(R_{25}), u'^{-1}u_6^{-2}(R_{26}), u'u_6^{-1}(R_{27})$ and $u'u_6^{-2}(R_{28})$ can be obtained from (B.38)-(B.43) by interchanging u and t.
II.

$a(t,u), b(t,u)$ and $C(t,u)$ are given by (B.25) and (B.28),

$$A(t,u) = \frac{(M_W^2+s+t+u)(t+u)}{2(s+t+u)}, \tag{B.44}$$

$$B(t,u) = \frac{(s+t+u-M_W^2)(su-t(s+t+u))}{2(s+t)(s+t+u)}, \tag{B.45}$$

$$S_1 = \int d\Omega_4\, u_5 = I_4^{(0,-1)}(A(t,u)), \tag{B.46}$$

$$S_2 = \int d\Omega_4\, u_5^{-1} = I_4^{(0,1)}(A(t,u),B(t,u),C(t,u)), \tag{B.47}$$

$$S_3 = \int d\Omega_4\, u_5^{-2} = I_4^{(0,2)}(A(t,u),B(t,u),C(t,u)), \tag{B.48}$$

$$S_4 = \int d\Omega_n\, t'^{-1}u_5^{-1} = \bar{I}_n^{(1,1)}(a(t,u),A(t,u),B(t,u),C(t,u)), \tag{B.49}$$

$$S_5 = \int d\Omega_n\, t'^{-1}u_5^{-2} = \bar{I}_n^{(1,2)}(a(t,u),A(t,u),B(t,u),C(t,u)), \tag{B.50}$$

$$S_6 = \int d\Omega_4\, t'u_5^{-1} = I_4^{(-1,1)}(a(t,u),b(t,u),A(t,u),B(t,u),C(t,u)), \tag{B.51}$$

$$S_7 = \int d\Omega_4\, t'u_5^{-2} = I_4^{(-1,2)}(a(t,u),b(t,u),A(t,u),B(t,u),C(t,u)), \tag{B.52}$$

$$S_8 = \int d\Omega_4\, t'^2u_5^{-1} = I_4^{(-2,1)}(a(t,u),b(t,u),A(t,u),B(t,u),C(t,u)), \tag{B.53}$$

The integrals $u'^{-1}u_5^{-1}(S_9), u'^{-1}u_5^{-2}(S_{10}), u'u_5^{-1}(S_{11}), u'u_5^{-2}(S_{12}), u'^2u_5^{-1}(S_{13})$ can be obtained from (B.49)–(B.53) by interchanging u and t.
III.

$$a(t,u) = \frac{2M_W^2u-(s+t+u-M_W^2)(s+t)}{2(s+t+u)}, \tag{B.54}$$

$$b(t,u) = \frac{(M_W^2-s-t-u)(s+t)}{2(s+t+u)}, \tag{B.55}$$

$A(t,u), B(t,u)$ and $C(t,u)$ are given by (B.44), (B.45) and (B.28) respectively,

$$T_1 = \int d\Omega_4\, u_5^{-1}u_6^{-1} = I_4^{(1,1)}(a(t,u),b(t,u),A(t,u),B(t,u),C(t,u)), \tag{B.56}$$

The integral $u_5^{-1}u_7^{-1}(T_2)$ is obtained from T_1 by interchanging t and u.

IV.
$a(t,u)$ and $b(t,u)$ are given by (B.54), (B.55) respectively,
$A(t,u), B(t,u)$ and $C(t,u)$ are given by (B.26), (B.27) and (B.28) respectively,

$$U_1 = \int d\Omega_4\, u_6^{-1}u_7^{-1} = I_4^{(1,1)}(a(t,u),b(t,u),A(t,u),B(t,u),C(t,u)) \tag{B.57}$$

V.
$a(t,u)$ is given by (B.25),

$$A(t,u) = \frac{(M_W^2-s-t-u)(s+u)}{2(s+t+u)}, \tag{B.58}$$

$$B(t,u) = \frac{(M_W^2-s-t-u)(s(s+t+u)-ut)}{2(s+t)(s+t+u)}, \tag{B.59}$$

$$V_1 = \int d\Omega_n\, u'^{-1}t'^{-1} = \tilde{I}_n^{(1,1)}(a(t,u),A(t,u),B(t,u)). \tag{B.60}$$

VI.

$a(t,u)$ and $B(t,u)$ are given by (B.25) and (B.45) respectively,

$$A(t,u) = \frac{(M_W^2-s-t-u)(t+u)}{2(s+t+u)}, \tag{B.61}$$

$$W_1 = \int d\Omega_n\, s_3^{-1} = \tilde{I}_n^{(0,1)}(A(t,u)), \tag{B.62}$$

$$W_2 = \int d\Omega_n\, t's_3^{-1} = \tilde{I}_n^{(-1,1)}(a(t,u),A(t,u),B(t,u)), \tag{B.63}$$

$$W_3 = \int d\Omega_n\, t'^2s_3^{-1} = \tilde{I}_n^{(-2,1)}(a(t,u),A(t,u),B(t,u)), \tag{B.64}$$

$$W_4 = \int d\Omega_n\, t'^{-1}s_3^{-1} = \tilde{I}_n^{(1,1)}(a(t,u),A(t,u),B(t,u)), \tag{B.65}$$

The integrals over $u's_3^{-1}(W_5), u'^2s_3^{-1}(W_6)$ and $u'^{-1}s_3^{-1}(W_7)$ can be obtained by interchanging u and t in (B.63)–(B.65).
VII

$a(t,u), b(t,u)$ are given by (B.54) and (B.55), $A(t,u), B(t,u)$ are given by (B.61) and (B.45), respectively

$$X_1 = \int d\Omega_n\, u_6^{-1}s_3^{-1} = \tilde{I}_n^{(1,1)}(a(t,u),b(t,u),A(t,u),B(t,u)), \tag{B.66}$$

$$X_2 = \int d\Omega_n\, u_6^{-2}s_3^{-1} = \tilde{I}_n^{(2,1)}(a(t,u),b(t,u),A(t,u),B(t,u)), \tag{B.67}$$

The integrals over $u_7^{-1}s_3^{-1}(X_3)$ and $u_7^{-2}s_3^{-1}(X_4)$ can be obtained by interchanging t and u in (B.66) and (B.67). Notice that the integrals R_6, R_{20} and W_1 do not appear in our final computer program. They only contain pole terms and are removed from the final result by mass factorization.

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C Figure Captions

Fig. 1. The lowest order Feynman diagrams contributing to the amplitude for the reaction $q(p_1) + \bar{q}(p_2) \to W(q) + \gamma(k)$. The solid, dashed and curly lines refer to the quark, W-boson and photon respectively.

Fig. 2. The order g_s^2 Feynman diagrams contributing to the amplitude for the reaction $q(p_1) + \bar{q}(p_2) \to W(q) + \gamma(k)$. The lines with loops refer to the gluon.

Fig. 3. The order g_s Feynman diagrams contributing to the amplitude for the gluon bremsstrahlung reaction $q(p_1) + \bar{q}(p_2) \to W(q) + \gamma(k) + g(k')$.

Fig. 4. The lowest order Feynman diagrams contributing to the reaction $q(p_1) + g(k') \to W(q) + q(p_2)$.

Fig. 5. Dalitz plots for the reaction $p + \bar{p} \to W(q) + \gamma(k) + g(k')$. Fig.a. shows $-U$ versus $-T$, where $(-T)^{(2)_{max}} = (-U)^{(2)_{max}} = S - (M_W^2 + \Delta)$, $(-T)^{(2)} = (-U)^{(2)} = S - M_W^2$. Fig.b. shows s_4 versus $-T$. Fig.c shows x_2 versus x_1, where $x_{1,m} = (M_W^2 - U)/(S + T)$, $x_1^* = (\Delta + M_W^2 - U)/(S + T)$, $x_{2,m} = (M_W^2 - T)/(S + U)$, $x_2^* = (\Delta + M_W^2 - T)/(S + U)$. The hatched areas represent the soft gluon parts of the Dalitz plots.

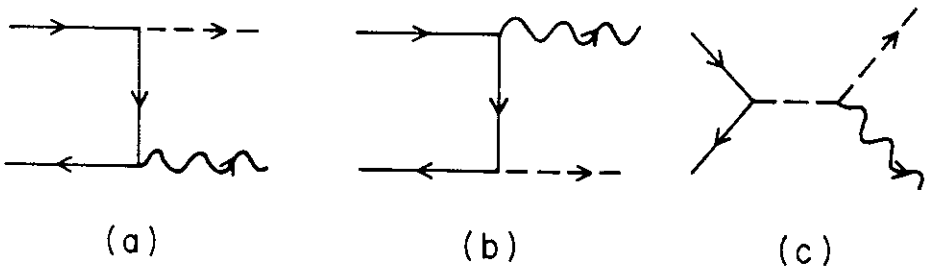
Fig. 6. $d\sigma/d\cos\theta_\gamma$ versus $\cos\theta_\gamma$ for the reaction $q + \bar{q} \to W^+ + \gamma$ at $\sqrt{s} = 95$ GeV.
solid line: Born approximation
dashed line : order α_s corrected cross section in mass factorization scheme C

Fig. 7. $(\sigma_{tot}^{W^+})^{-1} d\sigma/d\cos\theta_\gamma$ versus $\cos\theta_\gamma$ for the reaction $p + \bar{p} \to W^+ + \gamma + X$ at $\sqrt{S} = 0.63$ TeV.
solid line: Born approximation
dashed line : order α_s corrected cross section in mass factorization scheme C

Fig. 8. $(\sigma_{tot}^{W^+})^{-1} d\sigma/dE_\gamma$ versus E_γ for the reaction $p + \bar{p} \to W^+ + \gamma + X$ at $\sqrt{S} = 0.63$ GeV.
solid line: Born approximation
dashed line : order α_s corrected cross section in mass factorization scheme C

Fig. 9. $(\sigma_{tot}^{W^+})^{-1} d\sigma/d\cos\theta_\gamma$ versus $\cos\theta_\gamma$ for the reaction $p + \bar{p} \to W^+ + \gamma + X$ at $\sqrt{S} = 1.8$ TeV.
solid line: Born approximation
dashed line : order α_s corrected cross section in mass factorization scheme C
dashed dotted line: order α_s corrected cross section using the approximation in (8.2)

Fig. 10. $(\sigma_{tot}^{W^+})^{-1} d\sigma/dE_\gamma$ versus E_γ for the reaction $p + \bar{p} \to W^+ + \gamma + X$ at $\sqrt{S} = 1.8$ TeV.
solid line: Born approximation
dashed line : order α_s corrected cross section in mass factorization scheme C
dashed dotted line: order α_s corrected cross section using the approximation in (8.2)



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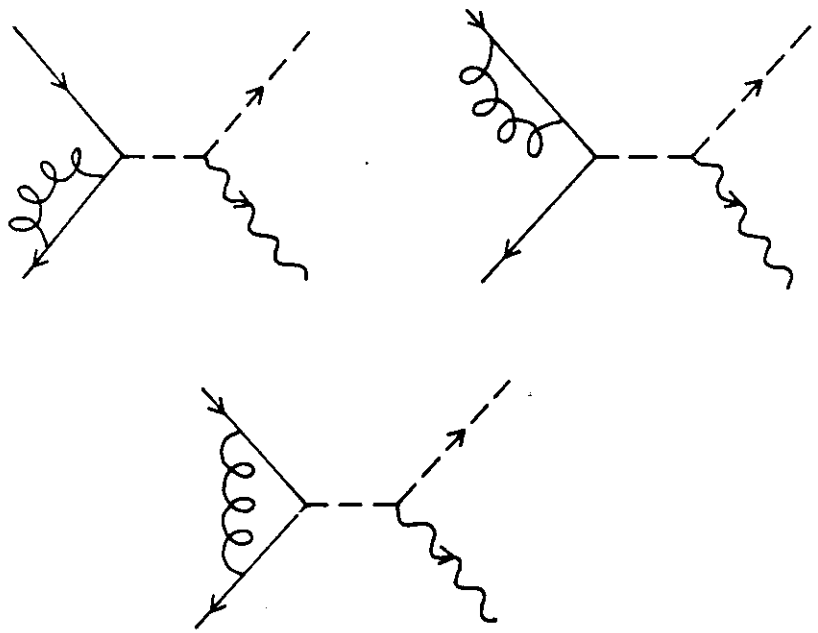

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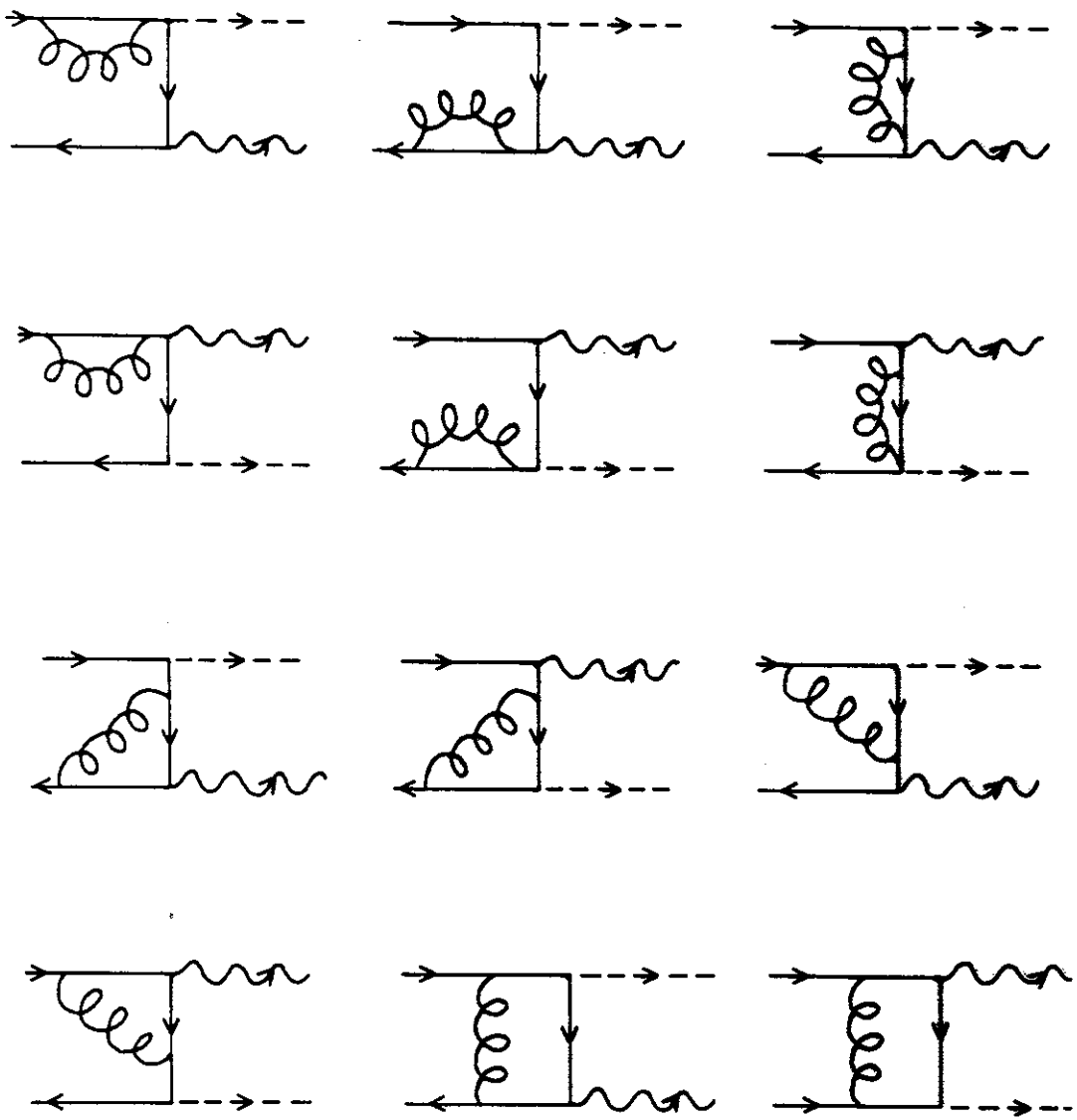
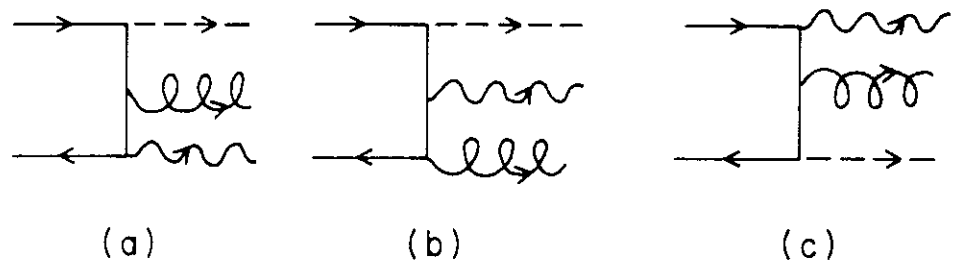
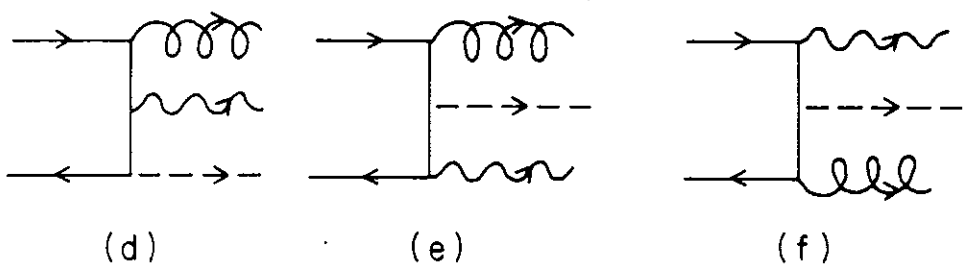
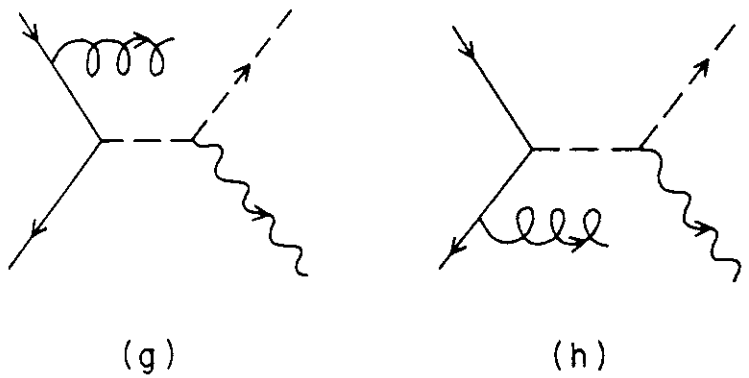


Fig. 2







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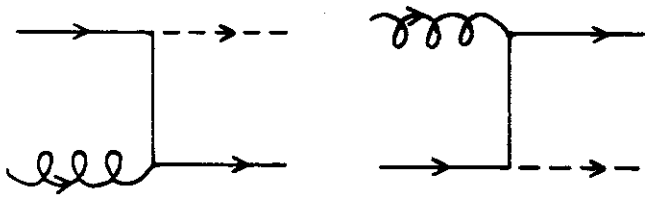

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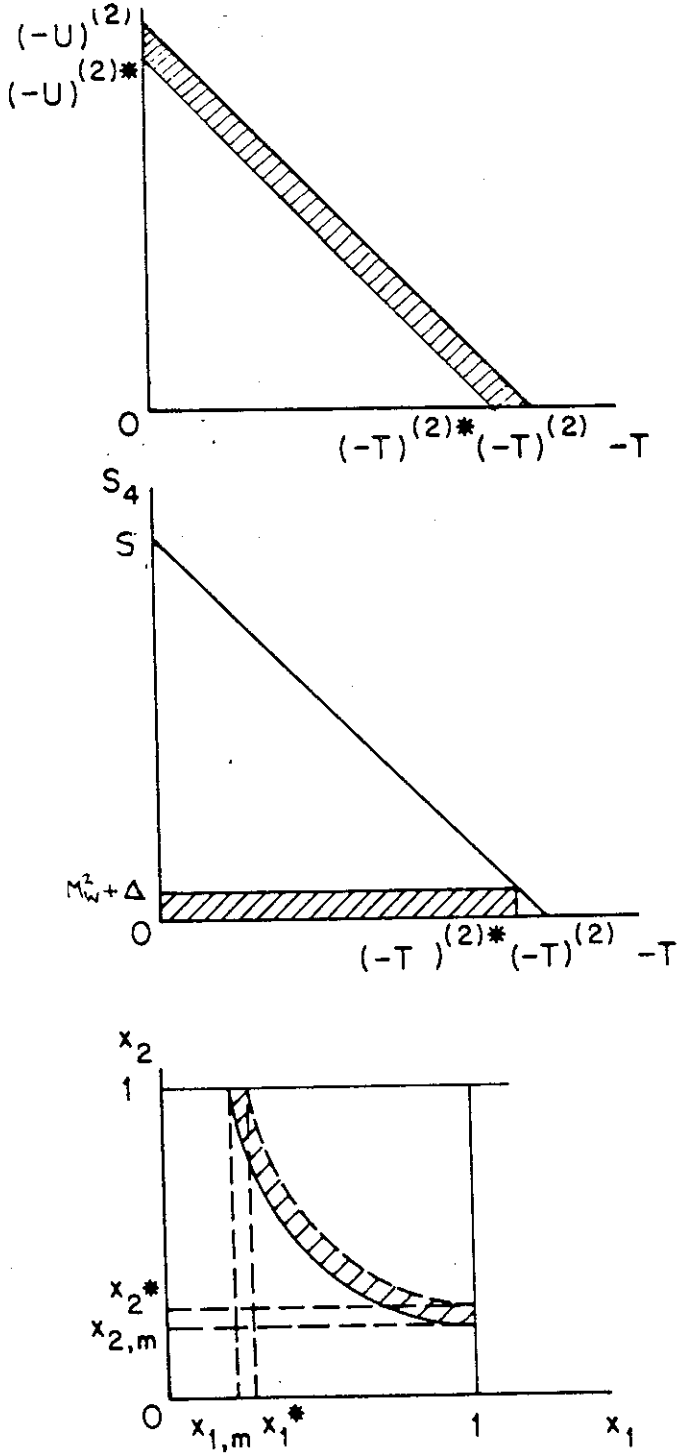

$(-U)^{(2)}$
$(-U)^{(2)*}$
0
$(-T)^{(2)*}$ $(-T)^{(2)}$ $-T$
S_4
S
$M_W^2+\Delta$
0
$(-T)^{(2)*}$ $(-T)^{(2)}$ $-T$
x_2
1
x_2^*
$x_{2,m}$
0
$x_{1,m}$ x_1^*
1
x_1

Fig.5

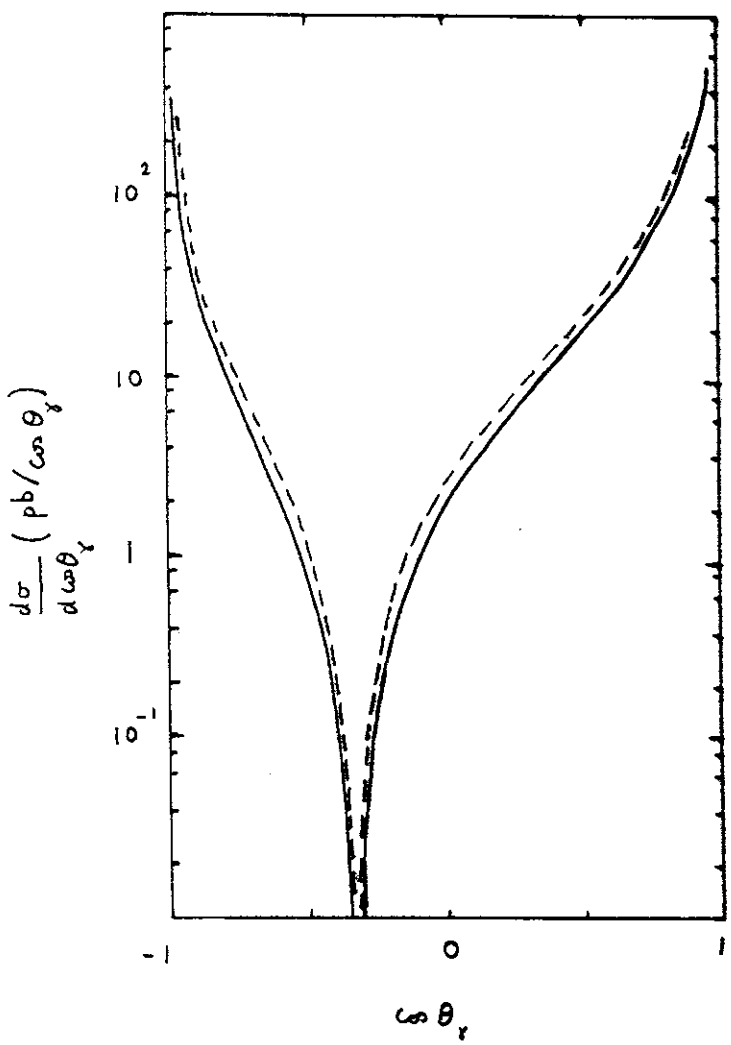

$\frac{d\sigma}{d\cos\theta_\gamma}$ (pb/$\cos\theta_\gamma$)
10^2
10
1
10^{-1}
-1
0
1
$\cos\theta_\gamma$

Fig.6

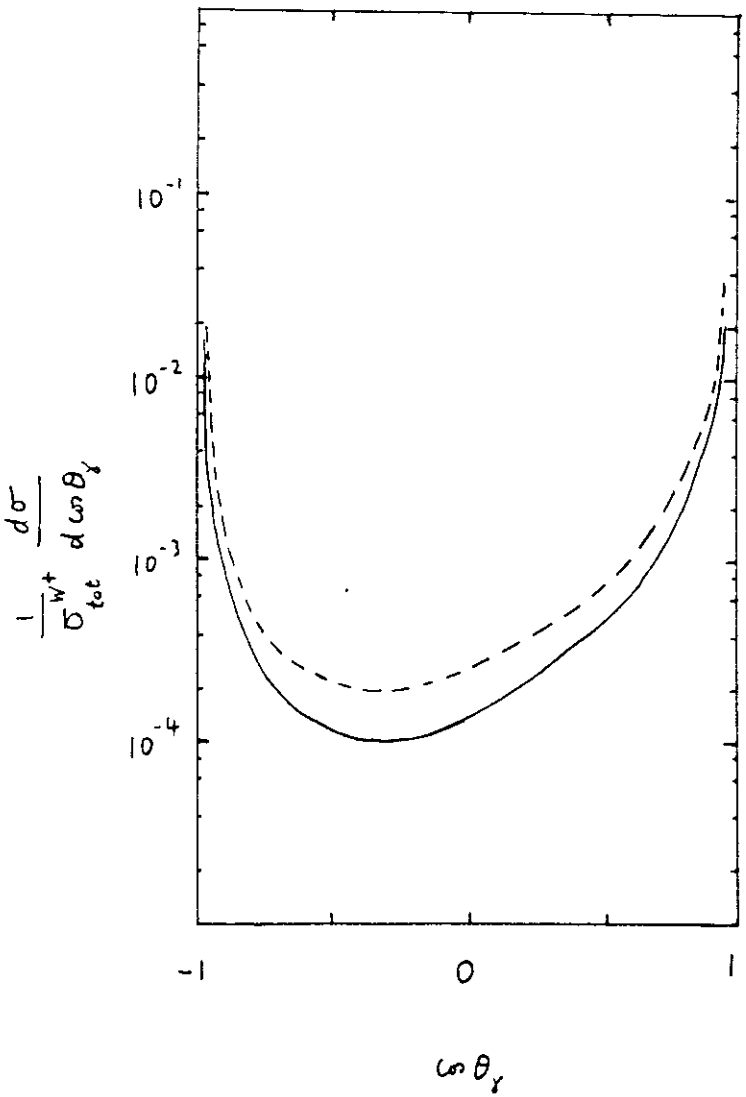

$\frac{1}{\sigma^{W^+}_{tot}} \frac{d\sigma}{d\cos\theta_\gamma}$
10^{-1}
10^{-2}
10^{-3}
10^{-4}
-1
0
1
$\cos\theta_\gamma$

Fig.7

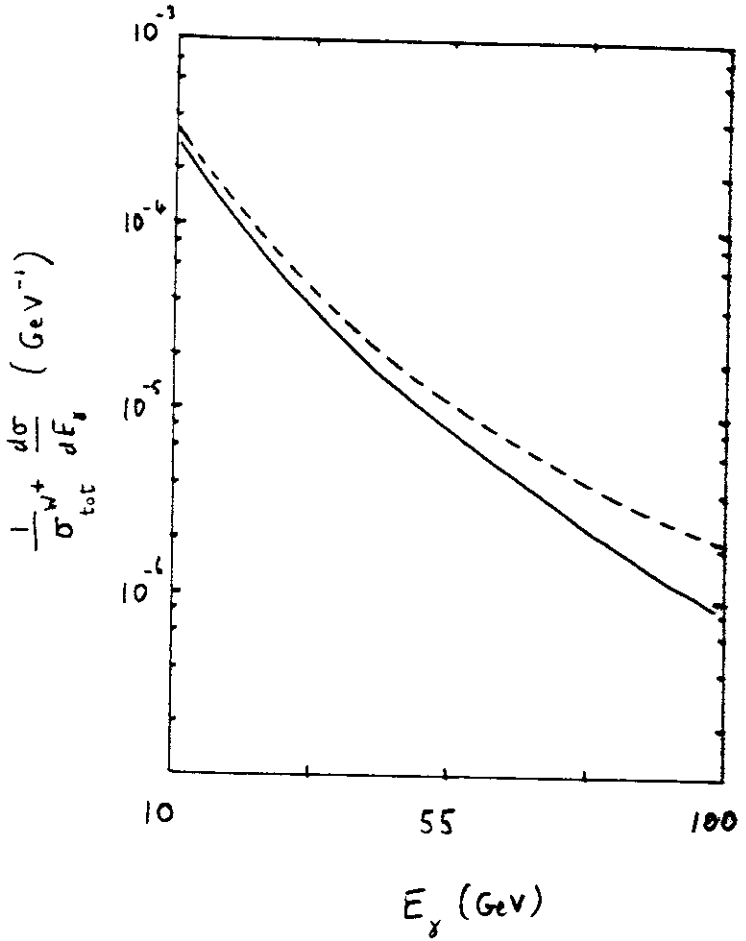
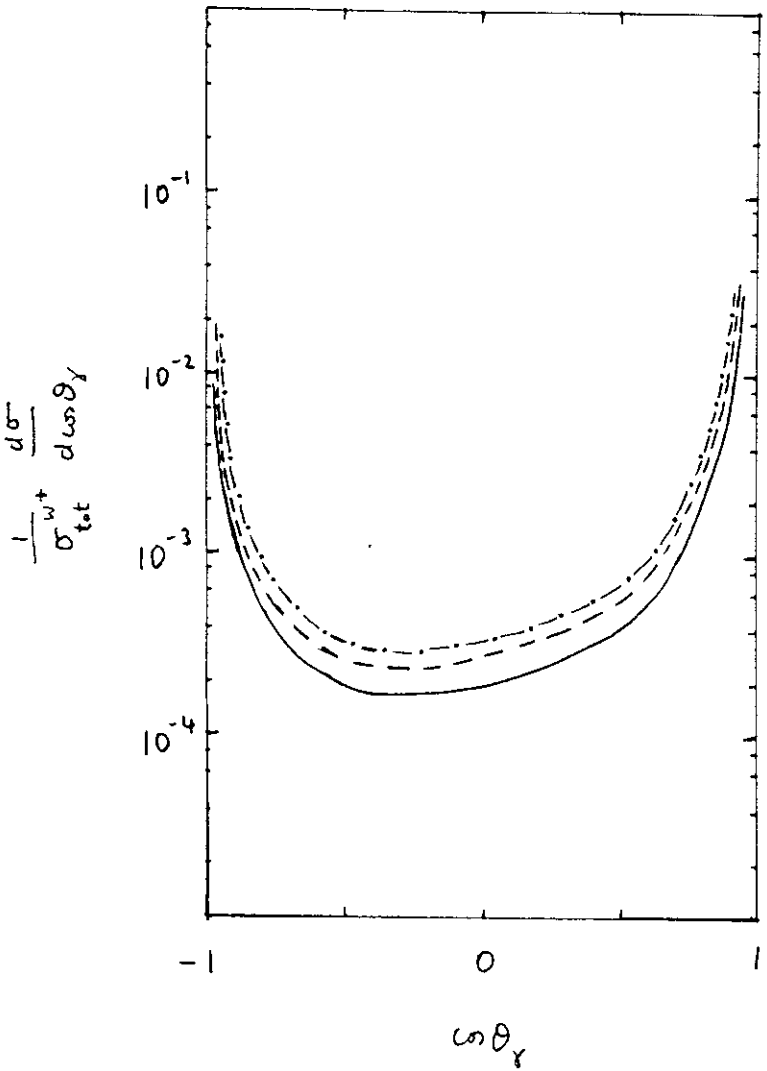

$\frac{1}{\sigma^{W^+}_{tot}} \frac{d\sigma}{dE_\gamma}$ (GeV^{-1})
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10
55
100
E_γ (GeV)

Fig.8


$\frac{1}{\sigma_{tot}^{W^+}} \frac{d\sigma}{d\cos\theta_\gamma}$
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10^{-2}
10^{-3}
10^{-4}
-1
0
1
$\cos\theta_\gamma$

Fig. 9

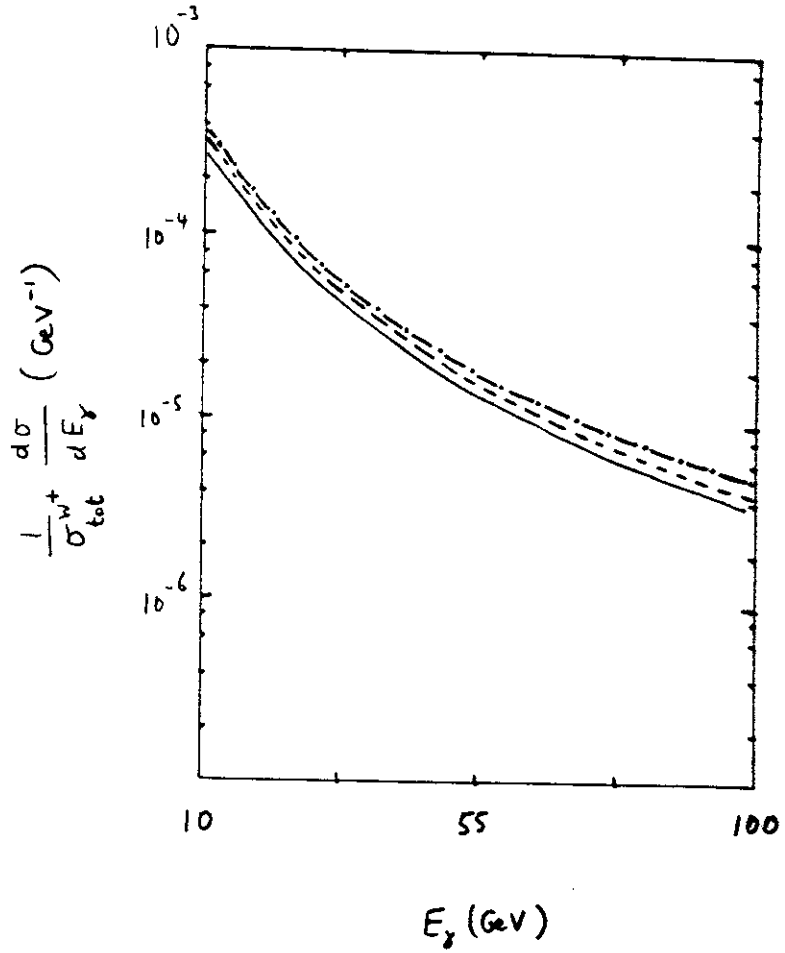
$\frac{1}{\sigma_{tot}^{W^+}} \frac{d\sigma}{dE_\gamma}$ (GeV^{-1})
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10
55
100
E_γ (GeV)

Fig. 10