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QCD Radiation off Heavy Particles[1]

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Abstract

An algorithm for an improved description of final-state QCD radiation is introduced. It is matched to the first-order matrix elements for gluon emission in a host of decays, for processes within the Standard Model and the Minimal Supersymmetric extension thereof.

The objective of this article is to summarize the improved description of parton shower evolution [1] recently introduced starting with Pythia 6.154 [2]. In particular, process-specific $\mathcal{O}(\alpha_{\mathrm{s}})$ matrix elements for gluon emission in decays $a \to bc$ are used to match the shower description to the correct emission rate in the hard-gluon region, and to provide the proper amount of 'dead cone' [3] suppression of collinear gluon emission off massive particles. The original motivation was to improve the understanding of $b\overline{b}$ events at LEP1. For linear colliders the applications to top, Higgs and SUSY physics are very important.

The traditional final-state shower algorithm [4] in Pythia is based on an evolution in $Q^2 = m^2$, i.e. potential branchings are considered in order of decreasing mass. A branching $d \to ef$ is then characterized by m_d^2 and $z = E_e/E_d$. For the process $\gamma^*/Z \to q\overline{q}$, the first gluon emission off both q and $\overline{q}$ are corrected to the first-order matrix elements

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for $\gamma^*/Z \to q\bar{q}g$. (The α_{s} and the Sudakov form factor are omitted from the comparison, since the shower procedure here attempts to include higher-order effects absent in the first-order matrix elements.)

This matching is well-defined for massless quarks, and was originally used unchanged for massive ones. A first attempt to include massive matrix elements did not compensate for mass effects in the shower kinematics, and therefore came to exaggerate the suppression of radiation off heavy quarks [5]. Now the shower has been modified to solve this issue, and also improved and extended better to cover a host of different reactions [1].

The starting point is the calculation of processes $a \to bc$ and $a \to bcg$, where the ratio

$$W_{\mathrm{ME}}(x_1, x_2) = \frac{1}{\sigma(a \to bc)} \frac{\mathrm{d}\sigma(a \to bcg)}{\mathrm{d}x_1 \, \mathrm{d}x_2} \tag{1}$$

gives the process-dependent differential gluon-emission rate. Here the phase space variables are $x_1 = 2E_b/m_a$ and $x_2 = 2E_c/m_a$, expressed in the rest frame of parton a. Using the standard model and the minimal supersymmetric extension thereof as templates, a wide selection of colour and spin structures have been addressed, exemplified by $Z^0 \to q\bar{q}$, $t \to bW^+$, $H^0 \to q\bar{q}$, $t \to bH^+$, $Z^0 \to \tilde{q}\bar{\tilde{q}}$, $\tilde{q} \to \tilde{q}'W^+$, $H^0 \to \tilde{q}\bar{\tilde{q}}$, $\tilde{q} \to \tilde{q}'H^+$, $\tilde{\chi} \to q\bar{\tilde{q}}$, $\tilde{q} \to q\tilde{\chi}$, $t \to \tilde{t}\tilde{\chi}$, $\tilde{g} \to q\bar{\tilde{q}}$, $\tilde{q} \to q\tilde{g}$, and $t \to \tilde{t}\tilde{g}$. The mass ratios $r_1 = m_b/m_a$ and $r_2 = m_c/m_a$ have been kept as free parameters. When allowed, processes have been calculated for an arbitrary mixture of "parities", i.e. without or with a γ_5 factor, like in the vector/axial vector structure of γ^*/Z. All the matrix elements are encoded in the new function `PYMAEL(NI,X1,X2,R1,R2,ALPHA)`, where `NI` distinguishes the matrix elements and `ALPHA` is related to the γ_5 admixture.

In order to match to the singularity structure of the massive matrix elements, the evolution variable Q^2 is changed from m^2 to $m^2 - m^2_{\mathrm{on-shell}}$, i.e. $1/Q^2$ is the propagator of a massive particle. Furthermore, the z variable of a branching needs to be redefined, which is achieved by reducing the three-momenta of the daughters in the rest frame of the mother. For the shower history $b \to bg$ this gives a differential probability

$$W_{\mathrm{PS},1}(x_1, x_2) = \frac{\alpha_{\mathrm{s}}}{2\pi} C_F \frac{\mathrm{d}Q^2}{Q^2} \frac{2\,\mathrm{d}z}{1-z} \frac{1}{\mathrm{d}x_1 \, \mathrm{d}x_2} = \frac{\alpha_{\mathrm{s}}}{2\pi} C_F \frac{2}{x_3 \, (1 + r_2^2 - r_1^2 - x_2)} \, , \tag{2}$$

where the numerator $1 + z^2$ of the splitting kernel for $q \to qg$ has been replaced by a 2 in the shower algorithm. For a process with only one radiating parton in the final state, such as $t \to bW^+$, the ratio $W_{\mathrm{ME}}/W_{\mathrm{PS},1}$ gives the acceptance probability for an emission in the shower. The singularity structure exactly agrees between ME and PS, giving a well-behaved ratio always below unity. If both b and c can radiate, there is a second possible shower history that has to be considered. The matrix element is here split in two parts, one arbitrarily associated with $b \to bg$ branchings and the other with $c \to cg$ ones. A convenient choice is $W_{\mathrm{ME},1} = W_{\mathrm{ME}}(1 + r_1^2 - r_2^2 - x_1)/x_3$ and $W_{\mathrm{ME},2} = W_{\mathrm{ME}}(1 + r_2^2 - r_1^2 - x_2)/x_3$, which again gives matching singularity structures in $W_{\mathrm{ME},i}/W_{\mathrm{PS},i}$ and thus a well-behaved Monte Carlo procedure.

Also subsequent emissions of gluons off the primary particles are corrected to W_{ME}. To this end, a reduced-energy system is constructed, which retains the kinematics of the branching under consideration but omits the gluons already emitted, so that an effective three-body shower state can be mapped to an (x_1, x_2, r_1, r_2) set of variables. For light quarks this procedure is almost equivalent with the original one of using the simple universal splitting kernels after the first branching. For heavy quarks it offers an improved modelling of mass effects also in the collinear region.

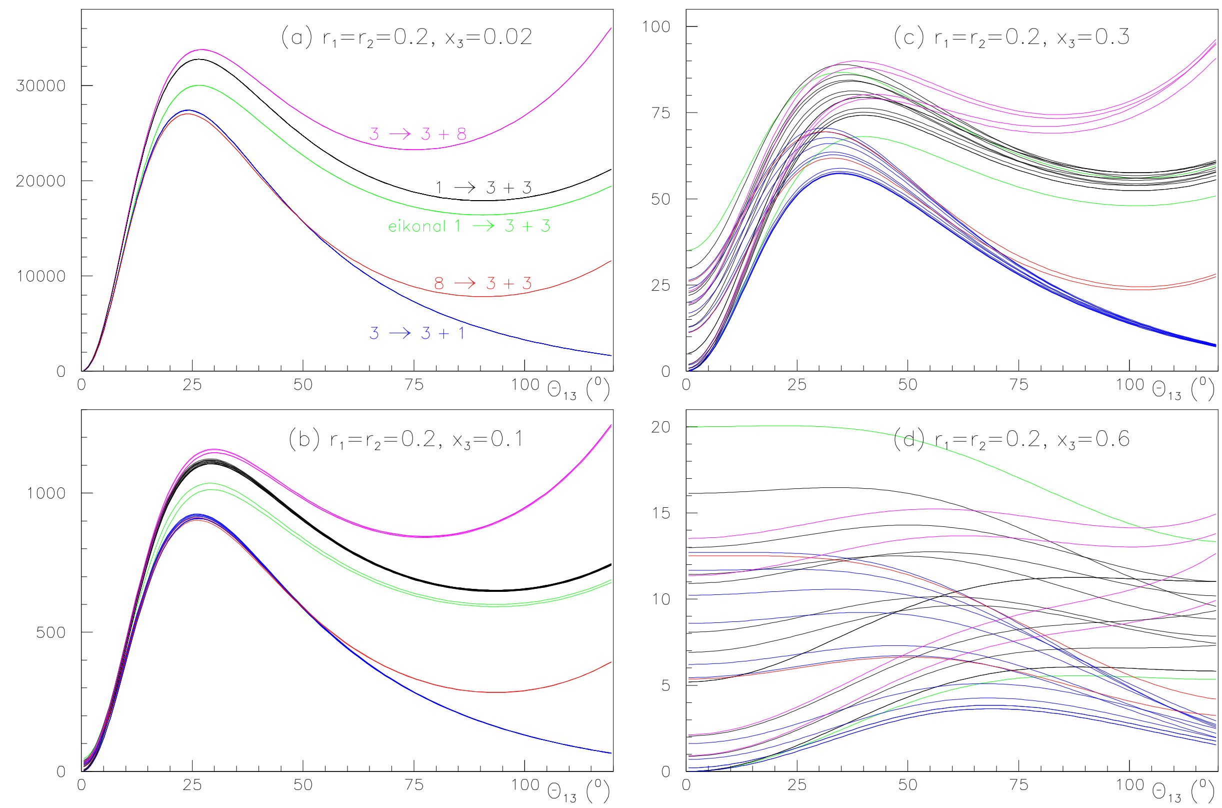

Figure 1: Gluon radiation pattern $W_{\mathrm{ME}}(x_1, x_2)$, eq. (1), as a function of the gluon emission angle, for four fixed gluon energy fractions, $x_3 = 2E_g/E_{\mathrm{CM}}$. The daughters/mother mass ratio is fixed at 0.2. The curves are for different combinations of colour and spin, and with/without a γ_5 factor where allowed.

Some further changes have been introduced, a few minor as default and some more significant ones as non-default options [1]. This includes the description of coherence effects and α_s arguments, in general and more specifically for secondary heavy flavour production by gluon splittings.

Further issues remain to be addressed, e.g. radiation off particles with non-negligible width. In general, however, the new shower should allow an improved description of gluon radiation in many different processes. Where it can be tested, for the amount of radiation off b quarks relative to light ones at LEP1, the new algorithm indeed is successful [1, 5].

As an illustration of the process dependence, Fig. 1 shows the radiation pattern of the various matrix elements calculated. In order to ease the comparison, the same fixed mass ratios have been used for all processes, $r_1 = r_2 = 0.2$. Furthermore, the large mass ratio highlights the dead cone effect, which shows a universal behaviour for small gluon energies. At large angles, and still small gluon energies, there is a dependence on the colour structure of the process, but not e.g. on the spin of the particles. This should be expected, since in the soft-gluon limit radiation can be described by a spin-independent eikonal expression [6]. Maybe more surprising is how completely this universality breaks down for more energetic gluons. Then processes are split not only by colour, but also by the spin structure, and the presence or not of a γ_5 in the matrix element, where allowed. (The figure only show the two extremes; by an arbitrary admixture of the two one would instead obtain a set of allowed bands.) Furthermore, the dead cone effect is shown to remain only for the case of a spin 0 particle decaying to two daughters also with spin 0. In retrospect, the process dependence is there also at small gluon energies, but is nonsingular and therefore invisible underneath the eikonal soft-gluon-singular contributions.

The above figure well illustrates that differences could be big in principle, but fortunately the reality is more forgiving. One reason is the big jump in mass between the b quark, on the one hand, and t, SUSY and any other potentially coloured particles, on the other. The most direct consequence is that the heavier particles typically generate only a small fraction of the total amount of QCD radiation, while b and lighter quarks produce the bulk of it. The b is light on the scale of the decaying particle, and so has a smaller dead cone than the one in Fig. 1.

A more realistic example of differences is then offered by a light Higgs state, say 115–130 GeV in mass as suggested by the MSSM scenario, decaying to $b\overline{b}$. The three-jet rate in such events typically is 10–15% higher than in $\gamma^*/Z^* \to b\overline{b}$ (or light quark) decays at the same energy. The difference is less for soft radiation, so the Higgs decay is only producing about 1% lower mean values for the b quark and B hadron fragmentation functions.

In $t\overline{t}$ events, the new algorithm increases the amount of radiation in the top production stage, but decreases it in the subsequent top decay. The difference is especially notable in the W hemisphere of the top decay, where the gluon emission rate is dropping rather steeper (with the angle away from the b quark) in the new program than in the old. This is related to a destructive interference between emission off the t and off the b in this hemisphere, while the older approach had its origins in $e^+e^- \to q\overline{q}$ events, where the interference is constructive. The net result is a small but visible decrease in the total amount of gluon radiation in $t\overline{t}$ events.

For supersymmetric processes, results largely depend on the actual masses. Assuming the charged Higgs mass to coincide with the $W^\pm$ one, the decays $t \to bW^+$ and $t \to bH^+$ give almost identical amounts of radiation. But if the stop mass agrees with the top one, there is more QCD radiation in the former production process than the latter. (While the difference in threshold behaviour here gives the opposite effect for ISR photon radiation, which can become more important.)

References

[1] E. Norrbin and T. Sjöstrand, LU TP 00–42 [hep-ph/0010012].

[2] T. Sjöstrand, *Computer Phys. Commun.* **82**, 74 (1994);
T. Sjöstrand, P. Edén, C. Friberg, L. Lönnblad, G. Miu, S. Mrenna and E. Norrbin, LU TP 00–30 [hep-ph/0010017], to appear in *Computer Phys. Commun.*

[3] Yu.L. Dokshitzer, V.A. Khoze and S.I. Troyan, *J. Phys.* **G17**, 1602 (1991).

[4] M. Bengtsson and T. Sjöstrand, *Phys. Lett.* **B185**, 435 (1987); *Nucl. Phys.* **B289**, 810 (1987).

[5] A Ballestrero et al., hep-ph/0006259, to appear in the *Proceedings of the LEP2 Monte Carlo Workshop.*

[6] G. Marchesini and B.R. Webber, *Nucl. Phys.* **B330**, 261 (1990).